United States Patent

Ladany

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[54] **DEVICE FOR THE GRAPHICAL SOLUTION OF SIMULTANEOUS EQUATIONS**

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[52] **U.S. Cl.** ....**235/61 B**

[51] **Int. Cl.** **G06c 3/00**

[58] **Field of Search**....235/61 B, 89 R, 61 GM, 85 R; 33/1 SD, 76 VA

[56] **References Cited**

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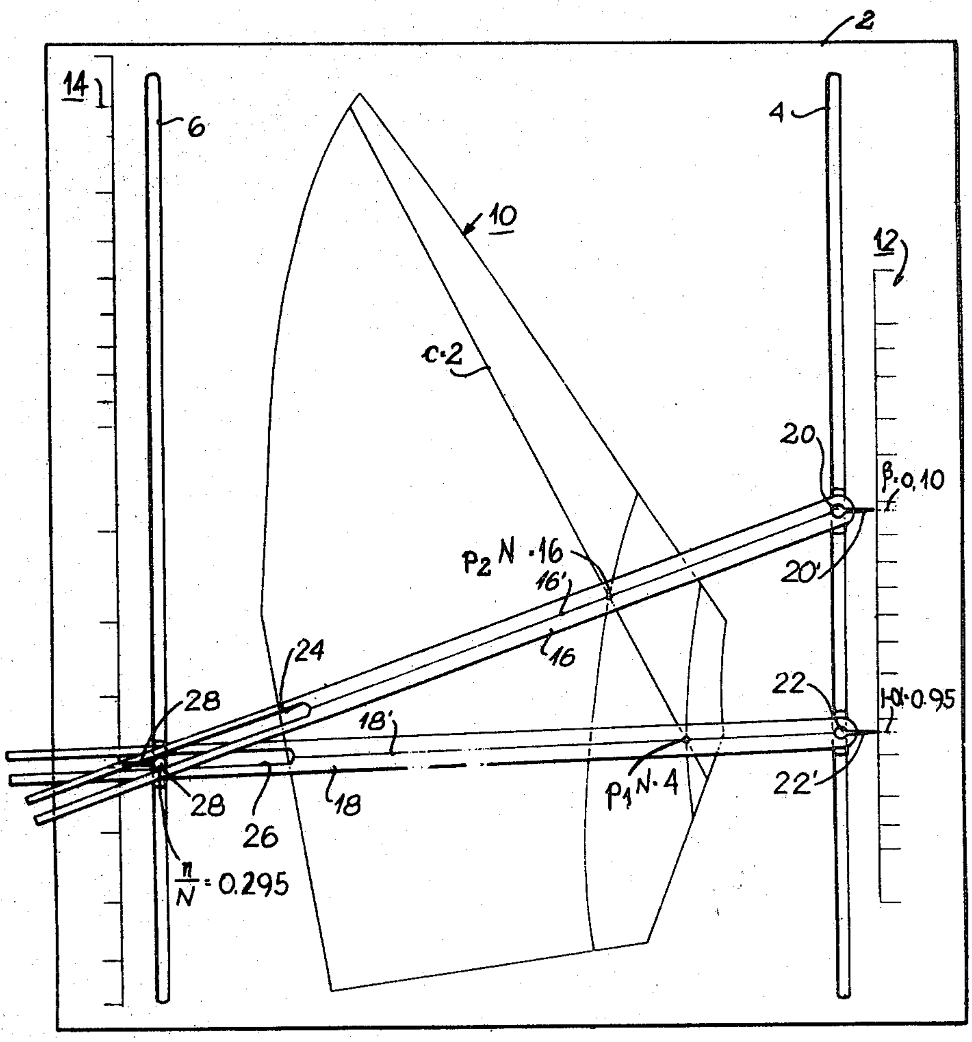
[57] **ABSTRACT**

A device for the graphical solution of the simultaneous equations $A = f(D, C, K)$, which equations can be represented by nomographs, e.g., Larson's Nomograph, having two spaced vertical axes bounding two families of curves. The device comprises a first straight line defining means (e.g., a transparent stick) having one end thereof presettable along the first vertical axis, a second straight line defining means (e.g., a second transparent stick) having one end thereof also presettable along the first vertical axis, their other ends intersecting at a common point on the second vertical axis, and means (e.g., a pin) disposed at the common point and movable along the second vertical axis.

8 Claims, 4 Drawing Figures



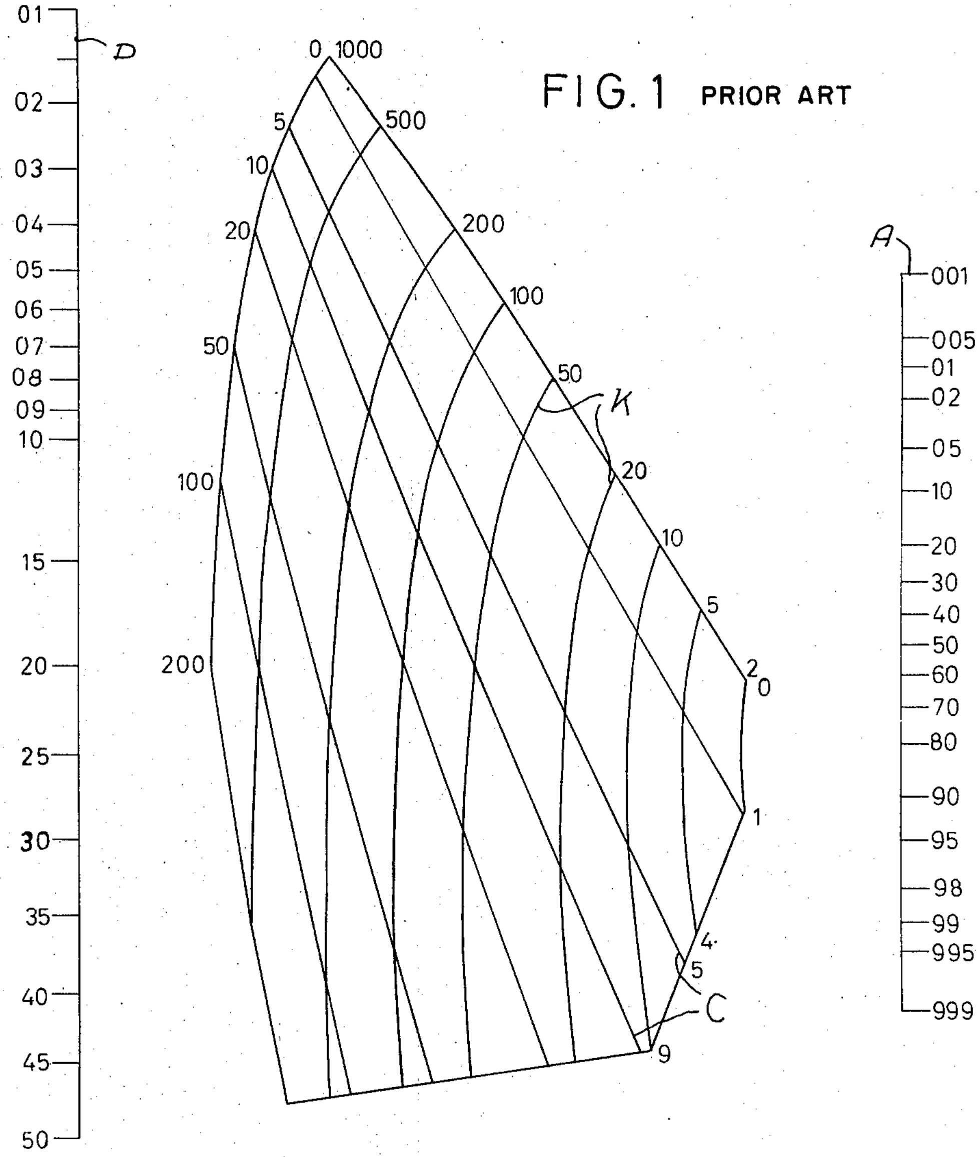


FIG. 1 PRIOR ART

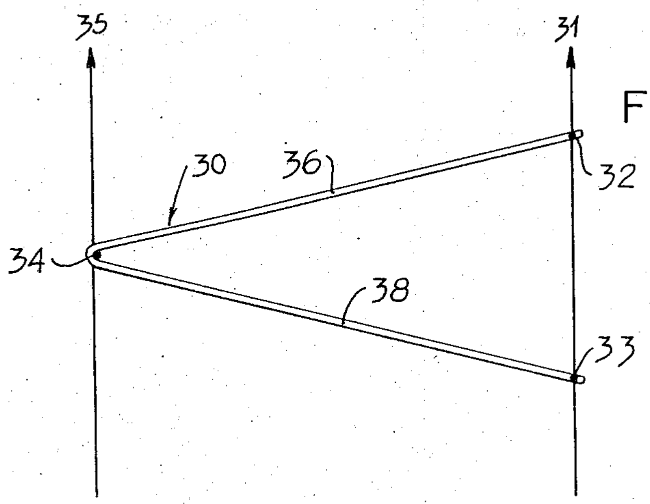


FIG. 3

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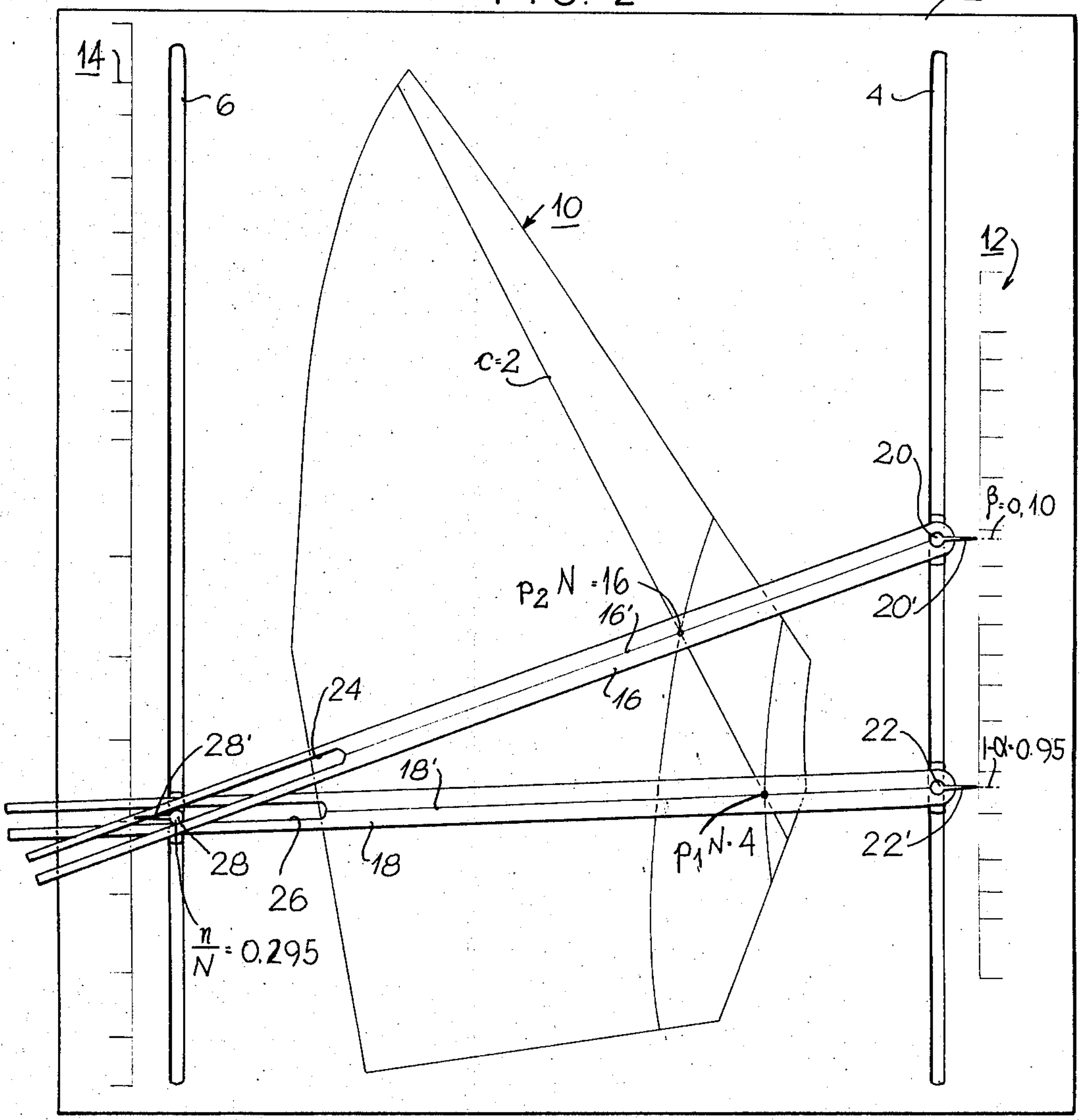
FIG. 2



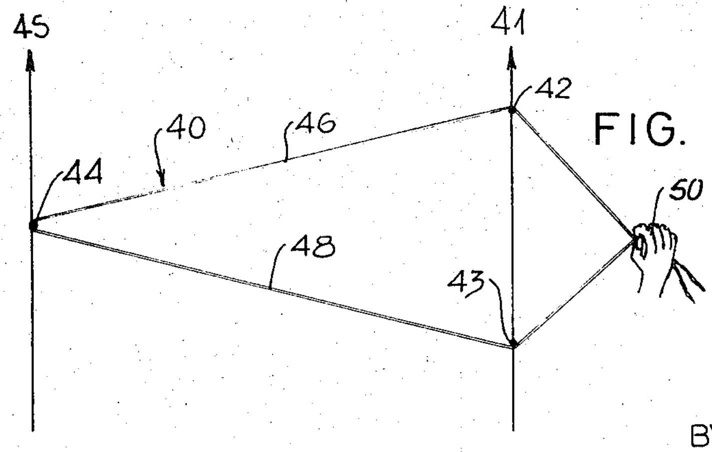


FIG. 4

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DEVICE FOR THE GRAPHICAL SOLUTION OF SIMULTANEOUS EQUATIONS

BACKGROUND OF THE INVENTION

The present invention relates to a device for the graphical solution of simultaneous equations, particularly of equations of the form $A=f(D, C, K)$ which can be represented by nomographs having two spaced vertical axes bounding two families of curves, one vertical axis being representative of A, the other vertical axis being representative of D, while C and K are constants each represented by a family of curves.

One such nomograph is in the field of statistical sampling plans and is called "Larson's Nomograph" of cumulative binomial distributions. Such nomographs are used for designing single-stage sampling plans. (See Larson, H.R., "A Nomograph of the Cumulative Binomial Distribution," *Industrial Quality Control*, Vol. 23, No. 6, December 1966, pp. 270–278.). The invention is particularly useful for designing sampling plans for individual lots when the lot size and sample size are large and the number of defectives in the lot is relatively small to the size of the lot, using Larson's Nomograph, and is therefore described below with respect to that application for purposes of example.

A single fraction-defective sampling plan for individual lots is designed for the conditions that a lot with a certain fraction defective p_1 will have a probability of rejection α, while giving protection to the consumer such that if the lot fraction defective is p_2, the probability of acceptance shall be β. Simultaneous solution of the cumulative hypergeometric equations (1) and (2) for n and c will yield the required plan.

$$\sum_{x=0}^{c} \frac{\binom{N-p_1N}{nx}\binom{p_1N}{x}}{\binom{N}{n}} = 1-\alpha \qquad (1)$$

$$\sum_{x=0}^{c} \frac{\binom{N-p_2N}{n-x}\binom{p_2N}{x}}{\binom{N}{n}} = \beta \qquad (2)$$

where

n = the lot size
n = the sample size
p_1N = the acceptable number of defectives in the lot
p_2N = the rejectable number of defectives in the lot
x = the number of defectives in the sample
c = the acceptance number

It should be noted that, because of the discreteness of the system, simultaneous solution of equations (1) and (2) will seldom yield values for n and c that exactly satisfy the values of $1-\alpha$ and β. Economic considerations will determine whether the risks obtained are satisfactory, or whether a different set of n, c values will be required.

It is not possible to obtain a direct simultaneous solution of equations (1) and (2). A trial-and-error method using tables has been given, but this is somewhat tedious to apply and is limited by the fact that the tables only apply for lot sizes no greater than 100 (with some scattered exceptions).

When the number of defectives in the lot is small relative to the size of the lot and sample, the Binomial II approximation can be applied to equations (1) and (2) as shown in equations (3) and (4).

$$\sum_{x=0}^{c} \frac{(p_1N)!}{x!(p_1N-x)!}\left(\frac{n}{N}\right)^x\left(1-\frac{n}{N}\right)^{p_1N-x} = 1-\alpha \qquad (3)$$

$$\sum_{x=0}^{c} \frac{(p_2N)!}{x!(p_2N-x)!}\left(\frac{n}{N}\right)^x\left(1-\frac{n}{N}\right)^{p_2N-x} = \beta \qquad (4)$$

Although these equations also can only be solved by trial and error, they have long constituted the easiest approach to the design of such sampling plans.

SUMMARY OF THE PRESENT INVENTION

The present invention provides a device enabling the rapid solution of these equations using the Larson Nomograph.

Larson's Nomograph of the cumulative binomial distribution is reproduced in FIG. 1. In the usual procedure of using this nomograph with binomial probabilities, the acceptable and rejectable probabilities of success are used directly with the corresponding values of $1-\alpha$ and β to determine the sample size and the acceptance number.

However, in applying the Binomial II approximation to the hypergeometric distribution it becomes clear that the value taken as the probability of occurrence n/N is unknown, because it is based on the as-yet unknown sample size. Further, the "number of trials" p_1N in equation (3) differs from the "number of trials" p_2N in equation (4). Nevertheless, in spite of these difficulties, which prevent direct solution of equations (3) and (4), a simple trial-and-error procedure can be used with the Larson Nomograph to yield the desired results.

The device of the present invention enables the desired results to be obtained in a very rapid manner from Larson's Nomograph.

Briefly, the device of the present invention provides means defining the two vertical axes of the Nomograph. The preferred embodiment of the invention utilizes a board having two spaced vertical slots, each including the appropriate scale markings appearing on the two vertical scales of the Nomograph. The device further includes two straight line defining means which, in the preferred embodiment, are in the form of transparent sticks each having a straight opaque line drawn thereon. One end of each of the sticks is presettable along one of the vertical axes, and the other ends of the two sticks intersect at a common point on the second vertical axis. Means are provided disposed at the common point and movable along the second vertical axis while the first and second line defining means are maintained straight. In the preferred embodiment, this is accomplished by making the other ends of the two sticks bifurcated, and providing a pin within the bifurcations and movable along the slot of the second vertical axis.

The specific procedure in which the device is used for obtaining the desired results from the Larson's Nomograph, is described below with respect to a specific example.

Also described below are variations in the novel device utilizing a rubber band or a piece of string, rather than the bifurcated sticks of the preferred embodiment.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention is herein described more particularly with reference to the drawings, wherein:

FIG. 1 illustrates Larson's Nomograph of the cumulative binomial distribution;

FIG. 2 illustrates a device constructed in accordance with the invention for yielding the desired results from Larson's Nomograph;

FIG. 3 illustrates another form of the present invention, utilizing a rubber band rather than rigid sticks; and

FIG. 4 illustrates a further form, utilizing a piece of string rather than rigid sticks.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

Larson's Nomograph, illustrated in FIG. 1, includes two families of curves, namely curves C representing the occurrence or the acceptance number ("*c*" in the earlier equations), and curves K representing the number of trials or sample size ("*n*" in the earlier equations). Further, it includes, a right-hand vertical scale A representing probability of "*c*" or fewer occurrences in "*n*" trials, and a left-hand vertical scale D representing the probability of occurrence in a single trial.

The device of FIG. 2 enables one to rapidly obtain the desired results form Larson's Nomograph of FIG. 1. It includes a board **2** formed with a pair of horizontally-spaced vertical slots **4,6**, the space **8** between the two slots being adapted to receive Larson's Nomograph, generally designated **10**. Vertical slot **4** carries scale markings **12** representing scale A of the Nomograph, and vertical slot **6** carries scale markings **14** representing scale B of the Nomograph.

The device further includes a pair of transparent sticks **16, 18**, each having an opaque line **16', 18'**, running along its longitudinal center. A pin **20** passes through an opening formed in one end of stick **16** and is movable in vertical slot **4**, and a second pin **22** passes through an opening formed in the corresponding end of stick **18** and is also movable in vertical slot **4**, Each pin carries a pointer **20', 22'**, cooperable with the scale markings **12**.

The opposite ends of sticks **16, 18**, are both bifurcated, as shown at **24, 26**. These ends are made to intersect at a point on vertical slot **6**, and a pin **28** movable in slot **6** passes through the intersecting bifurcated ends of the sticks. Pin **28** carries a pointer **28'** cooperable with scale markings **14**.

Pins **20, 22** are presettable at predetermined positions in vertical slot **4** at the beginning of the procedure, and once preset they are not intended to be moved during the remainder of the procedure. For this purpose, these pins may be of such dimensions to exert a significant friction when moved in slot **4**. Alternatively, they may each be formed with (e.g., threaded, or ball-and-detent) retaining elements. Pin **28**, however, is intended to be freely movable in slot **6**, and is therefore movable therein with a low coefficient of friction.

The device illustrated in FIG. 2 may be used for rapidly obtaining the desired information from Larson's Nomograph of FIG. 1 in the following manner:

In the Binomial II approximation, the probability occurrence is taken as n/N, and the numbers of trials are taken as p_1N and p_2N. These points are shown in FIG. 2.

The following example concerns the problem of finding the sample size and acceptance number for sampling from a lot of 200 items, given the following conditions: $p_1 = 0.02$, $\alpha = 0.05$, $p_2 = 0.08$, and $\beta = 0.10$. In such case, $p_1N=4$, and $p_2N=16$.

Pin **20** is preset in slot **4** to the value of β, namely 0.10 on scale **12**. Pin **22** is preset in slot **4** to the $(1-\alpha)$ position, namely 0.95, on scale **12**. Pin **28** is then moved along slot **6** until the lines **16'** and **18'**, on sticks **16** and **18**, respectively, are placed so that (as nearly as possible):

1. Line **16'** intersects the $p_2N=16$ curve;
2. Line **18'** passes through $p_2N=4$ curve; and
3. The intersecting points on both of the above curves lie on a single C-curve.

When these conditions obtain, the acceptance number "*c*" is read directly, and the sample size "*n*" is obtained by multiplying the value of n/N on the left-hand scale **14** by the lot size N. In the present example, $n=(200)(0.295)$, or 59; and $c=2$.

FIG. 3 illustrates a device utilizing a rubber band which may be used for following the same procedure as described above. In this variation, the rubber band, generally designated **30**, has one end fixed or presettable in a predetermined position along vertical scale **31**, e.g., by means of a pin **32**, and the opposite end of the rubber band is also fixed or presettable along scale **31**, e.g., by a second pin **33**. An intermediate portion of rubber band **30** is made to pass around a further pin **34** movable along a second vertical scale **35**. The rubber band is tensioned so that when pin **34** is moved the tension will maintain its two portions **36, 38** in straight lines. Portion **36** thus corresponds to straight lines **16'** of FIG. 2, and portion **38** corresponds to straight line **18'** of FIG. 2.

FIG. 4 illustrates a further variation of the device including a single piece of string. In FIG. 4, one end of the piece of string, generally designated **40**, passes around a pin **42** presettable along vertical scale **41**, and the opposite end of the string passes around a second pin **43** presettable along the same vertical scale. The intermediate portion of string **40** passes around a further pin **44** movable along vertical scale **45**. During this movement of pin **44**, tension is maintained in the string, e.g., by the user' hand **50**, so that string portions **46, 48** are maintained in straight lines and thus serve the same functions as lines **16', 18'** of FIG. 2.

Many variations, modifications, and other applications of the illustrated embodiments will be apparent.

What is claimed is:

1. A device for the graphical solution of the simultaneous equations $A=f(D, C, K)$, which equations can be represented by nomographs having two spaced vertical axes bounding two families of curves, one vertical axis representing A, the other vertical axis representing D, while C and K are constants each represented by a family of curves; the device comprising, means defining the first vertical axis and including scale markings representing the value of A, means defining the second spaced vertical axis and including scale markings representing the value of D, the two families of curves representing the constants C and K being disposable between said two vertical axes, first straight line defining means having one end thereof presettable along said first vertical axis, second straight line defining

means having one end thereof also presettable along said first vertical axis, the other ends of said first and second straight line defining means intersecting at a common point on said second vertical axis, and means disposed at said common point and movable along said second vertical axis while said first and second line defining means are maintained straight during said movement.

2. A device as defined in claim **1**, wherein said first and second straight line defining means are each in the form of a rigid, elongated member.

3. a device as defined in claim **2**, wherein each of said rigid members is a transparent stick having a straight opaque line drawn thereon.

4. A device as defined in claim **1**, wherein each of said first and second straight lines defining means is in the form of a rigid stick each pivotably mounted on said one end to a pin presettable along said first vertical axis, the opposite ends of said stick being bifurcated and received on a further pin movable along said second vertical axis.

5. A device as defined in claim **4**, wherein said device includes a board formed with a pair of vertical slots constituting said first and second vertical axes, said first-mentioned pins being presettable along said first vertical slot, and said further pin being movable along said second vertical slot while engaging said bifurcated ends of the sticks.

6. A device as defined in claim **1**, wherein said first and second straight line defining means are constituted

of a single rubber band, the opposite ends of which are fixable at any point on said first vertical axis, an intermediate point of the rubber band being passed around an element which is movable along said second vertical axis, the rubber band being tensioned to maintain in straight lines the two portions between said intermediate point and the two ends, as said element is moved along the second vertical axis.

7. A device as defined in claim **1**, wherein said first and second straight line defining means are constituted of a single piece of string, an intermediate point of which is passed around an element movable along said second vertical axis, the opposite ends of the string passing around two further elements presettable along said first vertical axis, the extremeties of said opposite ends being tensioned while said first-mentioned element is moved along said second vertical axis.

8. A device as defined in claim **1**, which device is used for the graphical solution of Larson's Nomograph of cumulative binomial distributions for designing single-stage attribute plans for individual small lots, the C-family of curves representing the number of occurrences, the K-family of curves representing the number of trials or sample size, said first vertical axis including scale markings representing the probability of C-occurrence or fewer occurrences in K-trials, and said second vertical axis includes scale markings representing the probability of occurrence in a single trial.

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