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RANQUE'S TUBE

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Since its discovery in Germany in 1945 by a Wartime Investigation Team, a vortex tube, which emits hot air from one end and cold air from the other when air under pressure is introduced tangentially near the middle of the tube, has been the source of speculation by refrigerating engineers concerning its practical applications and the cause of considerable controversy among heat transfer authorities in an effort to explain logically and mathematically the nature of the phenomena. In the February 1950 issue of REFRIGERATING ENGINEERING the article "An Analysis of the Hilsch Vortex Tube" was published. Some scientists and engineers disagreed with the theory of the tube's operation advanced in this article. In the accompanying article another theory is presented and a mathematical analysis is included in the Appendix. Readers are invited to comment on this analysis.

WHEN Georges Joseph Ranque stood before his colleagues of the Société Francaise de Physique in June 1933 and said that hot and cold air came out of a simple piece of pipe, he was received with skepticism.[1] He was a metallurgist associated with a steel company in the mountain town of Montlucon in central France; high-speed gas flow was out of his line.* No, said E. Brun, the aerodynamicist, stagnation temperature had been confused with static temperature and adiabatic wall temperature; the two streams weren't really cold and hot.[2] There is no further mention of the subject in the Société's Journal through 1944.

Ranque does not tell how he made his invention but it is possible he was concerned at some time with cyclone separators used to separate dust from air in steel plants. Air drawn from the center of such cyclones is slightly cool while air drawn from the outside is slightly warm. What would now be called the Ranque effect had been noticed, but apparently ignored, since such separators were first made.**

On December 12, 1931, Ranque filed a French patent docket on what is now known as the Ranque Tube, Hilsch Tube, Vortex Tube, or Vortex Refrigerator. After the French patent issued in 1932, he filed the same docket in the United States on December 6, 1932, and it issued on March 27, 1934.[3] He shows two basic types of design which may be called Counterflow and Uniflow, illustrated in Figures 1 and 2. He shows that the tangential entrance may consist of a single nozzle, a plurality of nozzles, or a set of blades. He describes how, by adjusting the size of the cold-air orifice or the restriction at the end of the hot tube, one may obtain a small quantity of very cold air or a larger quantity of moderately cold air. He mentions that the temperature of the hot tube reaches its maximum when the end of the hot tube is entirely closed, and that the more the pressure of the air supplied, the colder is the cold air. He speaks of having measured the pressure distribution inside the tube. Ranque must have come to hope he could accomplish significant things in refrigeration with his invention, for he assigns it to La Giration Des Fluides, or Whirl-Gas, of Montlucon, apparently a small company he had organized.

In attempting to broaden his claims, Ranque shows a model containing its own compressor, which he probably never tested. The compressor consists of a bladed rotor driven by an external motor. The air discharged from this rotor passes directly into stationary blades which guide the air tangentially into a uniflow tube. Not only would the rotor have to turn at an extreme speed but, as there is no cooling after compression, the cooler stream of air would still be warmer than the air supplied to the compressor.

The theory Ranque gives in the patent, later rejected, is as follows: The rotating gas spreads out in a thick sheet on the wall of the tube and the inner layers of this sheet press upon the outer layers by centrifugal force and compress them, thus heating them. At the same time the inner layers expand and grow cold. Friction between the layers is to be minimized, to which end the uniflow design is considered advantageous. The sheet is envisioned as having a rather sharp inner boundary, the center of the tube being filled with quiet gas.

The very different theory Ranque gave the Société will be taken up later. His remarks only amounted to 2½ pages in print, without illustrations or data, and he admits withholding further information for purposes of secrecy. The invention remained virtually unknown for more than 10 years. In view of the curiosity that demonstrations of the tube have invariably stirred up since 1946, it seems unlikely that Ranque made demonstrations. As to La Giration Des Fluides, it probably collapsed when it became clear that the device cannot compete with ordinary refrigeration machines. Ranque must have so lost interest by then that he was no longer inspired to publicize the invention. It is to be hoped that a full account of his work will be published.

In the spring of 1945, scientists sent to learn of wartime German developments found the tube being studied by Rudolf Hilsch, physicist, of the University of Erlangen. A working model was brought to the United States and this, combined with the publication of Hilsch's well known paper, started the present revival.[4] The writer learned of the tube on December 6, 1946, while associated with the General Electric Company, through A. B. Hubbard of that company. Interest in the tube now seems to be world-wide. It is a favorite subject for study in the schools and for demonstrations in the laboratory and shop, and engineers are still discovering it for the first time and temporarily entertaining hopes of revolutionizing the refrigeration industry, as did the writer in his turn.

Hilsch refers briefly to Ranque's paper of 1933 as the source of the idea, but it seems that he had not learned of the patent. He had arrived nevertheless at exactly the same design shown in certain of Ranque's drawings. Following Hilsch, nearly everyone has used similar designs to the complete neglect of the uniflow type so

* [illegible] publications of Ranque [illegible]
** [illegible]

far as the writer has learned. Hilsch mentions a hope of using the device for large-scale refrigeration such as the cooling of mines, but he has since become aware of the low efficiency.* He has actually used it in the liquefaction of gases.

Because Ranque's work has been virtually unknown, it has been widely assumed that Hilsch is the originator of the device, and the name "Hilsch Tube" has gained headway. It is hoped that this error will gradually be rectified, while Hilsch's contributions will still be appreciated.

Since 1946, writers of widely varying backgrounds have undertaken to explain the device, but with infrequent success. This may be attributed to the fact that while the device itself is extremely simple, the processes occurring in it are among the most difficult in gas dynamics. Few gas dynamicists giving attention to the problem have yet felt ready to publish their work. Nevertheless there is more sound theory in existence than is generally appreciated. Most notable is the work of Kassner and Knoernschild.[7] In *Part I* the writer presents a slightly different approach, less suited to prediction of the temperature change but intended to explain the mechanism simply. The work of K. & K. is briefly described.

The question of the possibility of commercial application is in need of clarification because no adequate analysis of the efficiency has appeared, and the enormous power consumption that would be required for most applications is not generally appreciated. This is dealt with in *Part II.*

SYMBOLS

- T static temperature, deg F abs
- p pressure, any units
- c_p specific heat at constant pressure, Btu per lb, F
- c_v specific heat at constant volume, Btu per lb, F
- k c_p/c_v
- J 778.26 ft-lb per Btu
- R gas constant, ft per F abs
- μ ratio of weight-rate of flow of cold gas to total flow
- η efficiency
- ln natural logarithm
- W total rate of flow, lb per sec
- a total availability, ft-lb per lb
- a_T availability due to temperature, ft-lb per lb
- a_p availability due to pressure, ft-lb per lb
- h enthalpy, Btu per lb
- s entropy, Btu per lb, F abs
- ρ density, slugs per ft³ (lb-sec² per ft⁴)
- r radius, ft
- θ angle, radians
- z axial distance, ft
- u, v, w velocity components in the r, θ, z directions, ft per sec
- ϕ velocity potential, ft² per sec
- ψ stream function, lb-sec per ft
- M Mach number = velocity / $\sqrt{kgRT}$
- ω angular velocity, rad per sec (not the rotation of fluid mechanics)
- μ absolute viscosity, lb-sec per ft²
- τ shear stress, lb per ft²
- C ωr^2, constant in a free vortex
- P rate of shear work, ft-lb per sec
- q rate of flow of heat, Btu per sec
- g 32.16 ft per sec²
- S surface area, ft²
- κ thermal conductivity, Btu per sec, ft, F
- N_{Pr} = $\mu g c_p/\kappa$, Prandtl number, dimensionless

Subscripts

- state of gas supplied to Ranque tube
- state of cold gas leaving Ranque tube
- body being cooled at fixed temperature
- state at end of reversible adiabatic expansion from 1 to p_2
- dead state—environmental temperature and pressure
- nozzle jet

Superscript

- * turbulent

PART I. THE MECHANISM

The present status of the theory, for both uniflow and counterflow types, may be summarized as follows: Fresh gas, before it has travelled very far in the tube, succeeds in forming an almost free vortex in which the angular velocity or rpm is low at the periphery and very high toward the center. But friction between the layers of gas undertakes to reduce all the gas to the same angular velocity, as in a solid body. This causes the inner layers to slow down and the outer layers to speed up as the gas moves along, amounting to the flow of work from the center to the outside of the vortex. At the same time, because the center of the vortex is much colder than the outside, heat flows toward the center, but not so rapidly as the work flows. The inner gas is originally cooled by its expansion; it stays partly cold by giving away its kinetic energy to the outer gas by friction without receiving as much heat energy in return. The outer gas in turn receives more kinetic energy than it loses heat energy, and this kinetic energy eventually becomes converted into internal energy through friction in the hot end of the tube. The flow is highly turbulent and the free vortex is supersonic.

The typical flow pattern in the counterflow type would be something like that shown in Figure 3. The solid lines show stream-surfaces of revolution about the axis. The position at which a particle enters determines its stream-surface, upon which it then describes a spiral path with ever-decreasing radius.** There is a null or stagnation point on the axis at O, and the stream-surface through this point divides what becomes the cold gas from what becomes the hot gas. The chief uncertainty about the flow pattern is the location of the null point, which will shift to the right and left as the flow of cold gas is decreased and increased, respectively. No data on this question are known to the writer but it is thought that Figure 3 may represent a reasonable estimate for equal hot and cold flows.

At every point in what Ranque calls the "chamber of action," extending from the orifice plate to, say, the null point, the centrifugal work flow exceeds the centripetal heat flow, resulting in a net outward flow of energy, shown by radial arrows. For this centrifugal energy flux to result in a decrease in stagnation temperature of the inner gas, it must cross the stream-surfaces. The stream-surfaces near the orifice plate, at [illegible], are mainly parallel to the radius, and therefore the gas in those stream-tubes only transfers energy within itself and emerges at the periphery of the discharge orifice scarcely cooled at all. This has been verified experimentally by bringing up a small tube to the mouth of the orifice and splitting the efflux into inner and outer parts. Under conditions where air was supplied at 105 psia and 70 F and 1/3 of the total flow issued through the cold-air orifice, the mixed stagnation temperature of the cold air was —30 F; when an outer annulus containing ¼ of the cold-air flow was separated, it had a stagnation temperature of 30 F while the remaining core had a stagnation temperature of —50 F. This diluting of the cold gas may be considered a disadvantage of the counterflow type since a separation at the orifice would always be necessary if one wishes to obtain the lowest possible temperature. In

* Private communication from Prof. Meissner, Munich, Sept. 1954.
** Actually, since the flow is turbulent, we can only speak of the mean paths.

the uniflow type this would be unnecessary.

Slightly downstream of the chamber of action, the gas has a graded stagnation temperature ranging from rather neutral at the axis to very high at the wall. That is why the tube wall in that region is hotter than the final mixed hot gas, and hotter than the tube wall either at the far end of the tube or at the inlet end of the tube.

Wall friction apparently has three beneficial, but small, effects. First, it removes some of the initial angular velocity of the outer layers, thus increasing slightly the initial value of the shearing friction. Second, it tends to keep the outer layers at a low angular velocity instead of permitting them to speed up as they receive energy from the inner layers, thus permitting the energy migration to proceed further. Third, it causes a small axial work flow down the hot tube by retarding the rotating mass downstream. This axial energy flux is indicated by the axial arrows in Figure 3.

The lowest possible temperature in the cold gas by the present theory would be that obtainable by a reversible and adiabatic expansion from the initial state to the final pressure:

$$\frac{T_0}{T_1} = \left(\frac{p_0}{p_1} \right)^{(k-1)/k} \qquad (1)$$

If the cold gas could give away all its kinetic energy without receiving any heat, it would emerge at this final temperature. So far about 1/2 of this temperature drop has been attained in the mixed cold gas, and about 4/7 in the core of the cold gas. Considering all the limitations of the actual process in the tube, it would appear that little further improvement could be expected. The process in the tube is highly irreversible.

Further details of the mechanism are given in the Appendix.

PART II. THE EFFICIENCY

The two common methods of heating—combustion and the passage of an electric current through a resistance—cannot be reversed. One places a higher value on refrigeration than on heating because refrigeration is so much harder to obtain. It is for this reason that in almost no case are we interested in using the hot gas from Ranque's tube. While it contains a good deal of thermodynamic availability due both to its temperature and to its pressure, there is usually no way to use these availabilities directly in the application, and to add auxiliary apparatus to recover them would so encumber the equipment that the original advantage of simplicity would be lost.

We must distinguish between two basic problems in refrigeration. On the one hand there is the problem of abstracting heat from a body always at a fixed temperature and pumping this heat out into the environment. The domestic refrigerator is an example of this problem. For this purpose a Carnot refrigerator is an ideal machine while a compression machine is a practical machine. A simple stream of cold gas is not suitable for this purpose; its use would involve irreversibility and inefficiency.

On the other hand there is the problem of cooling a substance down from environmental temperature to a certain low temperature. The cooling of air for air conditioning is an example of this problem. A proper machine for this purpose would be an isothermal compressor followed by an adiabatic expander. If a simple Carnot machine or a simple compression machine is used for this purpose, the efficiency of the process is only about ½.

If a machine which is ideal for one purpose is used as a criterion for a machine serving the other purpose, the criterion is not basic. We shall first look into some nonbasic criteria for Ranque's tube as a producer of cold gas, and then into a basic criterion.

Nonbasic Criteria

Carnot Criterion—We may define a coefficient of performance for Ranque's tube as equal to the heat taken out of the cold gas divided by the work of reversible isothermal compression:

$$\text{C.O.P.} = \mu \frac{J c_p (T_1 - T_c)}{RT_1 \ln \frac{p_1}{p_c}} \qquad (2)$$

where p_1 and T_1 are the pressure and temperature of the gas supplied to the tube, p_c and T_c are the pressure and temperature of the cold gas leaving the tube, c_p is the specific heat at constant pressure, R is the gas

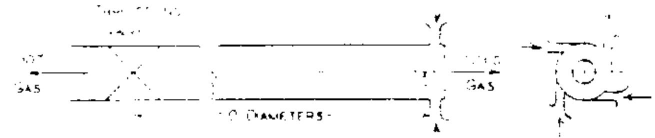

Fig. 1. The counterflow type.

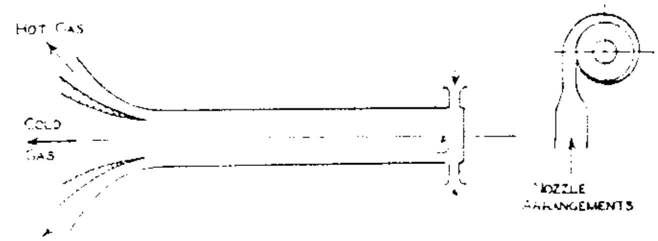

Fig. 2. The uniflow type.

constant of the gas, J is the mechanical equivalent of heat, and μ is the ratio of the weight-rate of flow of cold gas to the total flow.

The coefficient of performance of a Carnot refrigerator working between the temperatures T_c and T_1 is equal to $T_c/(T_1 - T_c)$. If we divide Equation (2) by this quantity, we obtain what may be called the efficiency of Ranque's tube on the Carnot basis:

$$\eta = \frac{J c_p (T_1 - T_c)^2 \mu}{RT_1 T_c \ln \frac{p_1}{p_c}} = \mu \frac{\left(1 - \frac{T_c}{T_1}\right)^2}{[illegible]} \qquad (3)$$

In the second of these forms, T_s is the temperature that would be reached in a reversible adiabatic expansion from p_1 and T_1 to p_c. We must be careful in interpreting the meaning of Equation (3).

Suppose we are interested in obtaining cold air at a temperature T_c. Then if, on the one hand, we use Ranque's tube to obtain that cold air and, on the other hand, we use a Carnot machine always working at T_c to obtain that cold air, Equation (3) gives the ratio of the works required by the two methods to produce the same amount of cold air, or the ratio of the amounts of cold air produced for the same work. Equation (3) is a meaningful criterion provided the possible competitor one wishes to consider can be no better than a machine of the fixed-temperature type, such as a Carnot machine or a conventional compression machine. Actually one would not be using the Carnot machine properly; it would be irreversibly and inefficiently applied. It turns out that an ideal machine would require only ½ the work of this inefficiently applied Carnot machine, for small temperature drops.

The efficiency of Equation (3) has been in the minds of several writers but for some reason it does not seem to have appeared in print. In Hilsch's small tube (Fig. 5) this efficiency reaches a maximum value of about 2.5 percent at 11 atmospheres pressure. For some reason

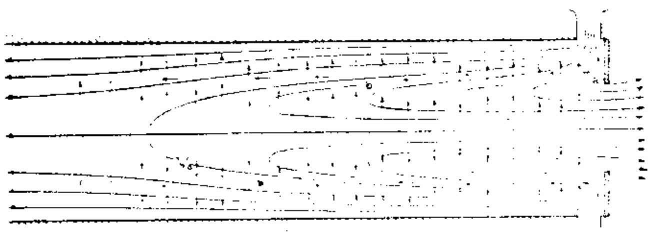

Fig. 3. Supposed flow pattern.

there has been an impression that this efficiency is over 10 percent. The efficiency of Equation (3) is approximately twice that of the basic criterion shown in Figure 6, and so from that figure one can perceive how it varies.

Criterion for Refrigeration at Fixed Temperature—Suppose we wish to use the cold air to remove heat from a body at a fixed temperature T_b. Then the cold air can only be utilized from T_c up to T_b, so that T_1 in the numerator of Equation (2) becomes T_b. Also T_1 in the Carnot C.O.P. becomes T_b. Then the efficiency becomes:

$$\eta_{\text{Fixed temp}} = \sigma \frac{J c_p (T_b - T_c)(T_1 - T_b)}{R T_c T_b \ln \frac{p_1}{p_0}} \quad (4)$$

This becomes zero if $T_b = T_c$ because then the cold air is not being utilized at all. It also approaches zero

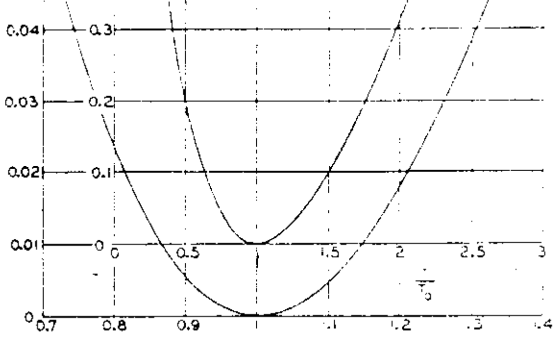

Fig. 4. Availability due to temperature of a perfect gas in steady flow.

as T_b approaches T_1 if T_c does not also approach T_1, because then the work of the Carnot machine vanishes. It tends to be a maximum when T_b is halfway between T_c and T_1, and at that condition it has roughly ½ the value of the basic criterion shown in Figure 6, or a maximum of around 0.6 percent. The approximate factor of 4 between Equations (3) and (4) when T_b is halfway between T_c and T_1 is due to the fact that in Equation (4) the Carnot machine is only working half as hard while at the same time only half the refrigeration in the cold gas is being utilized. The Carnot machine is now being used properly while the cold gas is being used improperly, giving a deficiency of a factor of about 2 compared with the basic criterion.

Turbine Criterion—Suppose the competitor against which we wish to compare Ranque's tube is a reversible and adiabatic turbine producing the same amount of cold gas σW at the same Temperature T_c, the work of the turbine being lost. This turbine will require a smaller pressure ratio than Ranque's tube, and also its compressor will only have to handle the flow σW instead of the flow W. The ratio of the work of the turbine's compressor to the Ranque tube's compressor turns out to be

$$\eta_{\text{Turbine}} = \sigma \frac{J c_p \ln \frac{T_1}{T_c}}{R \ln \frac{p_1}{p_0}} = \sigma \frac{\ln \frac{T_1}{T_c}}{\ln \frac{T_1}{T_2}} \quad (5)$$

where T_2 is the final temperature in a reversible adiabatic expansion from p_1 and T_1 to p_0.

This efficiency is rather insensitive to the pressure, unlike the efficiencies of Equations (3) and (4), and furthermore it is slightly higher at low pressures. In Hilsch's small tube (Fig. 5) at 11 atmospheres it has a maximum of about 13 percent at a σ of about 0.6, while at 2.5 atmospheres it has a maximum of about 15 percent at a σ of about 0.7. It is the loss of all the work from the turbine which permits Ranque's tube to show up much better in this criterion. Where a turbine is being used in this way, Ranque's tube is perhaps within a factor of 4 of competing, allowing for inefficiency in the turbine.

Hilsch's Criterion—It appears to the writer that what Hilsch calls the efficiency is the coefficient of performance of Equation (2). For the curves of Figure 5, at 11 atm this has a maximum of about 0.12 at a σ of about 0.6, at 7 atm and 4 atm a maximum of about 0.14 at a σ of about 0.65, and at 2.5 atm a maximum of about 0.13 at a σ of about 0.6. For a larger tube Hilsch makes calculations which seem to show values 40 percent to 60 percent higher.

The Basic Criterion

The basic criterion calls for comparing Ranque's tube against a reversible producer of cold gas, such as a reversible isothermal compressor followed by a reversible adiabatic expander. Let us make this comparison using the concept of availability.

The availability of a system is defined as the minimum amount of work required to bring the system from the dead state to the given state, operating in a large environment at a given temperature and pressure. It is also the maximum amount of work that can be obtained in reducing the system to the dead state. It is equal to the amount of work by any reversible process.

It is shown by Keenan[6] that the availability of a pure substance flowing steadily in the absence of changes in electricity, magnetism, capillarity, gravitational position, and velocity is given by

$$a = J\left[(h - h_o) - T_o(s - s_o)\right] \quad (6)$$

in which the subscript, $_o$, refers to the dead state—viz., that state in which the substance has the pressure of the environment, p_o, and the temperature of the environment, T_o.

For a perfect gas, Equation (6) reduces to

$$a = Jc_pT_o\left[\frac{T}{T_o} - 1 - \ln\frac{T}{T_o} + \frac{k-1}{k}\ln\frac{p}{p_o}\right] \quad (7)$$

In Equation (7), a has the units of ft-lb per lb. The last term represents the availability due to the pressure of the gas, and it will be recognized that this is simply

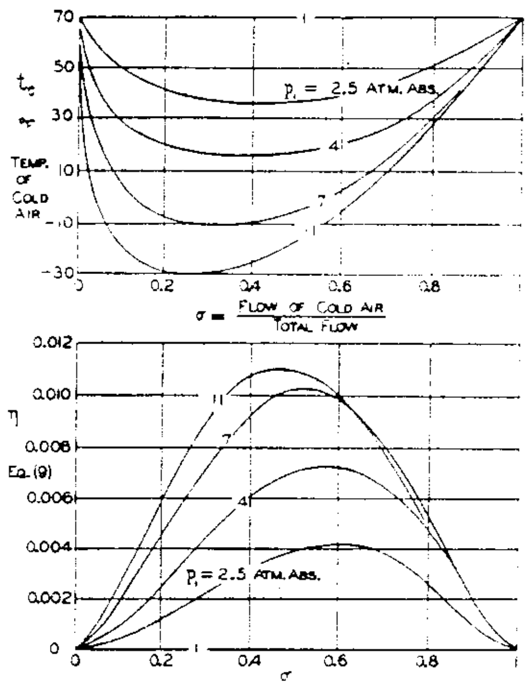

Fig. 5. (upper) Data of Hilsch for his small tube. Tube diameter 0.192 in., orifice diameter 0.0916 in., diameter of the single nozzle 0.0459 in., total flow at 11 atm 18.2 lb per hr or 4.12 cu ft per min requiring 0.623 hp for reversible isothermal compression. Cold air discharged at 1 atm abs.

Fig. 6. (lower) Basic efficiency as producer of cold air only from Figure 5 and Eq. (9).

the work of isothermal compression or expansion. If the pressure is lower than that of the environment, the last term is negative because work must be expended to pour the gas out into the environment.

The first three terms are a more interesting function; their sum represents the availability due to the temperature of the gas, and it is a property of this function that it has a positive value whether $T/T_o > 1$ or $T/T_o < 1$. That is because a heat engine can be operated using the gas either as a source of heat or as a sink, according to whether $T > T_o$ or $T < T_o$. The reversible process required can be accomplished by using an infinite series of Carnot engines, each operating at a slightly lower or higher temperature than the last, or much more simply by an adiabatic compression or expansion followed by an isothermal expansion or compression to restore the pressure. In Equation (7) the first two terms amount to $Jc_p(T - T_o)$; this is the heat exchanged with the gas. The third term is the amount of heat exchanged with the environment. The difference is the amount of work involved.

Figure 4 shows how the availability due to temperature varies. It is seen that for small temperature changes it is approximately proportional to the square of the temperature drop. Therefore a pound of gas at 10 deg F below the environment is worth approximately four times as much as a pound of gas at 5 deg F below the environment. In calculating the availability for small temperature drops, one is subtracting nearly equal numbers in Equation (7), and it is advisable to expand the logarithm in a series, giving

$$a_T = Jc_pT\left[\frac{1}{2}\left(\frac{T}{T_o} - 1\right)^2 - \frac{1}{3}\left(\frac{T}{T_o} - 1\right)^3 + \ldots\right] \quad (8)$$

We now define the efficiency of the Ranque tube as a producer of cold gas as equal to the work required by a reversible machine producing the same amount of cold gas at the same temperature divided by the work required by Ranque's tube using a reversible isothermal compressor. This gives

$$\eta = \frac{A}{A_1} = \frac{W_c a_c}{W_1 a_1} = \sigma\frac{a_c}{a_1} = \sigma\frac{\frac{T_c}{T_o} - 1 - \ln\frac{T_c}{T_o}}{\frac{k-1}{k}\ln\frac{p_1}{p_o}} \quad (9)$$

In Figure 5 are shown Hilsch's data for his small tube and in Figure 6 is shown the efficiency of this tube computed from those data using Equation (9). Since the air was actually somewhat humid, the availability was slightly higher than that credited to it. It is seen that the efficiency is in the neighborhood of 1 percent. For larger tubes under ideal conditions the efficiency may reach 2 percent.

If the hot gas is credited as to temperature availability, the efficiency of the tube of Figure 5 is of the order of 2 percent; if the hot gas is credited as to temperature and pressure availability, the efficiency ranges between 10 percent and 20 percent.

Conclusion

To air condition a passenger automobile requiring 7000 Btu per hr of refrigeration, assuming a coefficient of performance of 0.13, would require an ideal horsepower of 21. This would call for a larger tube which, also operated at a somewhat higher pressure, might bring the actual horsepower to a minimum of 20. For a domestic refrigerator requiring 400 Btu per hr, using the same assumptions, the ideal horsepower is 1.2.

V. Shaefer and I. Langmuir, in their celebrated work on the artificial causing of snow, have found that the ice crystals in the cold air act as very good seeds for the snow. Where a gas is throttled to obtain liquid by the Joule-Thomson effect, the Ranque tube would produce more liquid per pound of gas supplied to it. Some of the liquid would collect on the tube wall near the nozzles and should be drained off through proper openings before it is re-evaporated. However, in most cases where such throttling is done on an appreciable scale, the products pass through heat exchangers which maintain the process, and it would produce no advantage thermodynamically to use the tube.* Hilsch uses the tube in a small gas liquefier in place of an expansion engine or other preliminary cooling means; advantages of rapidity of starting are cited.[7]

While Ranque's tube is finding a few isolated uses, we have seen that it cannot serve in installations of size where power consumption is a consideration. For refrigeration in minute quantities or for very occasional use where stored compressed air is available, it is suitable. It remains one of the most remarkable inventions of the century.