In the state of darkness, relative to the invisible aethereal medium, existing at the close of the 18th century, *Laplace* doubtless considered it sufficient to deal with expressions which give the forces acting on the planets, without inquiring into the geometrical nature and physical mechanism involved in the generation of these forces, which were then believed to lie beyond the reach of the investigator.

After the development of *Faraday*'s Experimental Researches in Electricity, and *Maxwell*'s mathematical interpretation of these results, very different views came to be entertained by geometers and natural philosophers. Yet it was only the developments brought out in the »Electrod. Wave-Theory of Phys. Forc.«, which seemed to justify definite expectations of forming clear geometrical and physical conceptions of the mechanism involved in the action of the magnetic and the planetary forces across space. Recently these conceptions have been verified and extended, and therefore we shall here attempt to give a geometrical and physical interpretation of the potential which so long proved bewildering to the physical mathematician.

In the »Electrod. Wave-Theory of Phys. Forc.«, 1917, p. 134, it is pointed out that if waves be the basis of physical action across space, then the amplitude of such waves when propagated spherically and without resistance, in tridimensional space, will be given by the equation:

$$A = k/r\,. \tag{26}$$

In an address to the Academy of Sciences of St. Louis, Sept. 21, 1917, I gave this simple formula and pointed out its geometrical and physical significance. Professors *F. E. Nipher*, *E. A. Engler* and other physicists were present and showed great interest in the results announced, from which it would appear that this law had largely or entirely escaped the notice of earlier investigators.

Now by comparing this expression (26) with that in (25) above, we notice that the wave amplitude has the same form as the potential defined by *Laplace* in 1782. The question thus arises: Can the coincidence in form be due to chance, or is the potential in fact an analytical expression for the total aether stress due to the superposition of waves from all the atoms, each of the waves being of the average wave amplitude, appropriate to the coordinates in the field of force about an attracting mass? To get at the truth in this interesting inquiry, we notice that *Laplace*'s formula of 1782 integrates the mass of every particle of the attracting body, divided by its distance, which corresponds to a summation of the effects due to the superposed wave amplitudes and thus increases directly as the mass, each set of waves superposed from the atoms in any element $\sigma\,dx\,dy\,dz/r$, being independent of all the rest, but the triple integral including the accumulated wave action of the whole mass:

$$V = M/r =$$
$$= \iiint \{\sigma / \sqrt{[(x-x')^2+(y-y')^2+(z-z')^2]}\}\,dx\,dy\,dz\,. \tag{25}$$

The elements under the integral signs represent the individual potentials of every particle, and thus the potential increases directly as the mass whose wave-effects are integrated. This conforms rigorously to our conceptions of the *Newton*ian law of attraction, and involves no approximation, since the element of mass $dm = \sigma\,dx\,dy\,dz$ can be made so small as to apply to every single particle or atom.

At first sight the mere fact that the potential V as thus defined follows the law of wave amplitude in tridimensional space strikingly suggests that the wave-theory represents the order of nature. To find out by exact calculation what is the probability of such a coincidence occurring by mere chance, we may proceed as follows.

Taking the expressions for two independent curves, the amplitude and the potential, we have:

$$A = y = k/x, \quad V = y = M/x\,. \tag{27}$$

It will be noticed that they belong to the same geometrical species — both being rectangular hyperbolas referred to their asymptotes — and can be made identical throughout, from $x = 0$ to $x = \infty$, by introducing a summation Σ, such that $\Sigma k = M$.

Accordingly it appears that by the mere variation of a parameter the curves are made to coincide rigorously, point by point, from $x = 0$ to $x = \infty$. Therefore the chances against such a rigorous coincidence accidentally occurring throughout infinite space, $x = 0$ to $x = \infty$, becomes infinity to one, or,

$$C = \int_0^\infty dx = \infty \tag{28}$$

and thus its actual occurrence points unmistakably to a true law of nature.

It seems therefore certain and incontestible that the potential represents geometrically and physically the total accumulated stress due to the whole mass under the average wave amplitude of the field about the attracting body in question.

It is to be noticed also that physically our definition of the potential confirms this conclusion. In free space there is no cause to alter the spherical distribution of the waves, as they expand with increase of r. But in or near the shadows of the earth, as shown in the »Electrod. Wave-Theory of Phys. Forc.«, a circular refraction of the sun's waves will necessarily occur. The sun's potential varies, even at a constant distance, near the shadow of the earth; and owing to this refraction, fluctuations of the moon's motion should arise near the time of lunar eclipses, as fully explained in this work of 1917. This circular refraction of the electrodynamic waves in passing through the earth's mass changes the potential or total accumulated stress due to the integration of the waves from all the atoms, under the average wave amplitude and distribution of the waves in the space near the shadow of the earth: and therefore also the sun's forces acting on the moon.

Partially released from the sun's control, by the interposition of the body of the earth, with its refractions of the sun's wave-field, the moon tends to fly the tangent while traversing the region of the shadow cone, and thus arise the fluctuations of the moon's mean motion, connected with lunar eclipses, which long perplexed *Laplace*, *Hansen*, *Newcomb*, *Hill*, *Brown* and other astronomers.

8. Explanation of the Propagation of the Wireless Waves around the Earth.

In the unpublished manuscript sent by the writer to the Royal Society in November, 1914, which was the first