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Power conversion of energy fluctuations

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An analysis of the theoretical efficiency of converting the nonlinear equilibrium thermal fluctuations of a system to a useful form of energy is made based on equilibrium distributions of statistical mechanics. For the model considered, electric energy fluctuations, as in a resistive material, are converted to a useful form of energy by combining these fluctuations with those from a nonlinear resistive material at another temperature. The system selected for the computed results uses two Alkemade diode models, each in contact with a heat reservoir, as the resistive materials. The resulting rectified current is calculated by deriving a master equation giving the probability of discrete electron jumps across the potential barriers of the diodes. From this, the efficiency is computed of the direct power conversion of thermal to electric energy and of electric to thermal energy as a function of the output or input voltage of the system. These results for the examples are in agreement with fluctuation theory and show that the conversion efficiency increases as the system size decreases to make the fluctuation energy a significant fraction of the total energy. The results also show that there are no theoretical limits on the assumed model in approaching the efficiency limitations of the Carnot cycle. Finally, the limitation on the available fluctuation power that could be provided from a unit area of a resistive film to an assemblage of small circuits was considered. It was determined that this very high limitation was of the order of 10^8 W/m^2. This indicated the potential that, if the material and fabrication problems could be solved, this technique could yield practical amounts of power at high efficiencies over a wide range of temperatures.

INTRODUCTION

The fluctuations of the energy of a system that is in thermal equilibrium with a heat reservoir has been extensively analyzed theoretically and observed experimentally. The present paper reports a process by which the energy of the fluctuations about the mean energy can be converted to a useful form of energy or can be used in a refrigeration cycle. The theoretical method to be used in a study of this conversion process is based on the concepts of statistical mechanics. These concepts are useful in analyzing the behavior of systems for which the energy of the fluctuations about the mean are a significant fraction of the total energy. For this investigation, a physical model was chosen for which the fluctuation energy is observed as electric energy fluctuations in a resistive material or across a potential barrier. This fluctuation energy is converted to useful direct power by impressing the voltage fluctuations across a nonlinear resistor that is maintained at a different temperature.

This concept is closely related to the older concept of using thermionic emission to convert thermal to electric energy. Since the concept of the thermionic energy converter was first suggested in 1915,[1] much creative research has been reported on techniques for increasing the efficiency of the thermionic energy conversion process.[2-5] Preliminary inconclusive experimental and theoretical work on the concept of the present paper was started in 1947 at the Radiation Laboratory of the University of California. The theoretical work was based on the Nyquist theorem[6] for computing the noise power from the linear heated resistive material and a nonlinear diode and the results gave higher conversion efficiencies than the Carnot cycles for certain limiting cases as these model sizes decreased. It was necessary to have a healthy skepticism about the results for these limiting cases until corroborated by more complete computations and experimental results.

After the work of van Kampen on nonlinear thermal fluctuations in diodes[7-11] it was possible to use his results for computing the conversion efficiency of this process using a simpler model. For the model chosen it was found that the conversion efficiency was limited to the Carnot efficiencies.[12,13] Since then additional theoretical and experimental work on the thermodynamics of the nonlinearity in diodes has been reported,[14] but the results leave open the question of the application of the laws of thermodynamics to this conversion process when small systems are considered. Since there is now an increased interest in new approaches to producing useful power and since recent developments in manufacturing techniques give promise that it may soon be feasible to produce the desired microscopic-size circuits for this process,[15-17] it can now be useful to give the theoretical results for simple models of this process that can be compared with experimental results.

PROBABILITY DISTRIBUTION FOR DIODE MODEL

In the analysis of van Kampen of nonlinear thermal fluctuations, the diode model introduced by Alkemade[11] was used. For this idealized diode model it is assumed that the electron transit time is negligible and that there is no correlation between the electrons emitted from the electrodes of different metals with different work functions. It is also assumed that the total diode system is in equilibrium with a temperature bath. For this simple model, the computation will follow the approach used by van Kampen to derive a probability distribution based on the probability of discrete electron jumps across the potential barrier of a single Alkemade diode. In order to use this approach, both the resistance at the higher temperature T_r and the nonlinear resistance at the lower temperature T_c in the circuit shown in Fig. 1 will be represented by diodes R_r and R_c, respectively. Later the results for a model where R_r is a linear resistor will be discussed. The terminals across the battery voltages V_1 and V_2 are the output power terminals for the circuit in a direct power conversion mode or are the input power terminals for the circuit in a direct refrigeration mode. To simplify the notation, let C be the sum of the diode capacities C_r and C_c and let V be the sum of the battery voltages V_1 and V_2 and the contact potential differences of the diodes.

The master equation for the probability distribution $P(N)$ can be derived as a function of the number of electrons N in excess on the upper side of the condenser representing the total combined capacitance C of the two diodes. A and G are the saturation currents from the surfaces of the higher work functions and B and D are the currents emitted from the surface of lower work functions. The electrons in the B and D currents have emission probabilities that are modified by the changes in electrostatic energies as given by the following transitions:

$$(e^2/2C)[N^2-(N-1)^2]=(e^2/C)(N-\tfrac{1}{2}), \tag{1}$$

$$(e^2/2C)[(N+1)^2-N^2]=(e^2/C)(N+\tfrac{1}{2}). \tag{2}$$

Then, following van Kampen, the master equation is

$$\frac{\partial P(N)}{\partial t}=A\,P(N+1)+B\exp\left(-\frac{e^2(N-\frac{1}{2})}{kT_rC}\right)P(N-1)-A\,P(N)-B\exp\left(-\frac{e^2(N+\frac{1}{2})}{kT_rC}\right)P(N)+G\,P(N-1)+D\exp\left(\frac{e^2(N+\frac{1}{2})}{kT_cC}\right)P(N+1)-G\,P(N)-D\exp\left(\frac{e^2(N-\frac{1}{2})}{kT_cC}\right)P(N). \tag{3}$$

The values of the currents of the diodes are determined by the work functions of the electrodes, the battery potential across the diodes, and the temperatures of the diodes. Using the substitutions

$$\alpha=e^2/kT_rC,\quad \beta=e^2/kT_cC,\quad K=A/G, \tag{4}$$

and letting

$$A=B\,e^{\alpha n},\quad G=D\,e^{\beta m},$$

where

$$n=(C_c/e)V,\quad m=(C_r/e)V,$$

so that

$$V=(e/C)(m+n), \tag{5}$$

the master equation becomes

$$\frac{1}{G}\frac{\partial P(N)}{\partial t}=KP(N+1)+K\exp[-\alpha(N-\tfrac{1}{2}+n)]P(N-1)-KP(N)-K\exp[-\alpha(N+\tfrac{1}{2}+n)]P(N)+P(N-1)+\exp[\beta(N+\tfrac{1}{2}-m)]P(N+1)-P(N)-\exp[\beta(N-\tfrac{1}{2}-m)]P(N). \tag{6}$$

Using the operator F defined by

$$Ff(N)=f(N+1),\quad F^{-1}f(N)=f(N-1), \tag{7}$$

the master equation may be simplified to

$$\frac{1}{G}\frac{\partial P(N)}{\partial t}=\left((F-1)\{K+\exp[\beta(N-\tfrac{1}{2}-m)]\}-F^{-1}\{1+K\exp[-\alpha(N+\tfrac{1}{2}+n)]\}\right)P(N). \tag{8}$$

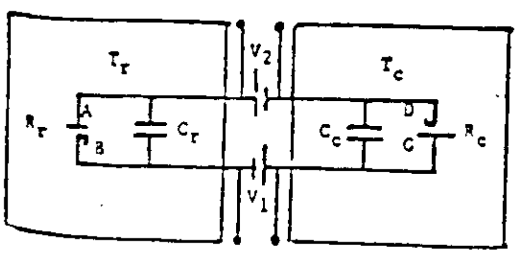

Fig. 1. Circuit for power conversion of fluctuation energy from two diodes at different temperatures.

For the equilibrium state, since $\partial P(N)/\partial t = 0$,

$$(K + \exp[\beta(N - \tfrac{1}{2} - m)])$$
$$- F^{-1}\{1 + K\exp[-\alpha(N + \tfrac{1}{2} + n)]\}) P_{N} \qquad (9)$$

must be a constant. Also P_{N} must vanish as $N \to \infty$. Therefore, we can write for this equilibrium state

$$P(N) = \frac{1 + K\exp[-\alpha(N - \frac{1}{2} + n)]}{K + \exp[\beta(N - \frac{1}{2} - m)]} P(N-1). \qquad (10)$$

This relation enables the probability of N excess electrons on the upper side of C to be given in terms of $P(0)$ as

$$P(N) = \left(\prod_{j=1}^{N} \frac{1 + K\exp[-\alpha(j - \frac{1}{2} + n)]}{K + \exp[\beta(j - \frac{1}{2} - m)]}\right) P(0). \quad (11)$$

From this expression for $P(N)$ it can be easily shown that as K increases to make

$$K \gg \exp[\beta(N - \tfrac{1}{2} - m)],$$

then

$$P(N) \simeq \exp[-\tfrac{1}{2}\alpha N(N + 2n)] P(0). \qquad (13)$$

This Gaussian distribution for N in this limiting case is, as it must be, similar to the distribution derived by van Kampen for the single heated diode where the center of the distribution is displaced by the dc potential en/C across the heated diode R_r.

If $n = m = 0$, and $\alpha = \beta$ for the two diodes of Fig. 1, then it can be seen from Eq. (11) that

$$P(N) = \exp(-\tfrac{1}{2}\alpha N^2) P(0). \qquad (14)$$

This Gaussian distribution centered at the origin is an expected result since this model includes the symetric model of two identical diodes at the same temperature that are connected as in Figure 1 with V_1 and V_2 being the contact potential of each diode so that $V_1 = -V_2$ for identical diodes.

The recursion equation when R_e is reversed can be written

$$P(N) = \frac{K\exp[-\alpha(N - \frac{1}{2} + n)] + \exp[-\beta(N - \frac{1}{2} - m)]}{K + 1} \times P(N-1) \quad (15)$$

For these two diodes in parallel it can be easily seen that for $\alpha = \beta$ and $m = -n$

$$P(N) = \exp[-\tfrac{1}{2}\alpha N(N + 2n)] P(0), \qquad (13)$$

which, of course, is the same distribution as the van Kampen result for the equivalent diode to the two diodes in parallel.

RECTIFIED CURRENT IN POWER CONVERSION AND REFRIGERATION MODES

The recursion equation for $P(N)$ can be used to compute the rectified current in the power conversion and refrigeration modes. The expected rectified current $I(N)$ for each value of N in the equilibrium state for the model of Fig. 1 is

$$I(N) = D\exp[\beta(N - \tfrac{1}{2})] P(N) - G P(N-1). \qquad (16)$$

Using the relation between $P(N)$ and $P(N-1)$ of Eq. (10) gives

$$I(N) = GK \frac{\exp[(\beta - \alpha)(N - \frac{1}{2}) - (\beta m + \alpha n)] - 1}{1 + K\exp[-\alpha(N - \frac{1}{2} + n)]} \qquad (17)$$

and for the total direct current I_t we have

$$I_t = \sum_{N} I(N). \qquad (18)$$

It can be seen from this equation that if $T_r = T_e$ and $V = 0$ that no direct current will flow. Therefore, no power can be obtained from this model without there being a temperature difference between diodes. This result holds for all circuits of diodes that were analyzed. In particular, this result holds for the model for Eq. (15) where the sense of diode R_e is reversed from that of Fig. 1. For this model, the current $I(N)$ is given by

$$I(N) = GK \frac{\exp[(\beta - \alpha)(N - \frac{1}{2}) - (\beta m + \alpha n)] - 1}{K\exp[(\beta - \alpha)(N - \frac{1}{2}) - (\beta m + \alpha n)] + 1} P(N). \qquad (19)$$

From this equation it can also be seen that for $V = 0$ and $T_r = T_e$ no direct current will flow.

This result is of interest in determining if the diode open circuit voltages analyzed by McFee in a paper on self-rectification in diodes[13] can be used as a power source. He raises the important question of whether there is a conflict between the van Kampen diode theory and the second law of thermodynamics and he argues that the expected voltage across the diode can, for small diodes, exceed the contact potential. This question of possible conflict with the second law will not be resolved here but it is related to this same question to point out that, as can be seen from Eqs. (17) and (19), a model for R_r and R_e consisting only of Alkemade diodes does not give a self-rectification current and no power can be obtained from these models when the diodes are at the same temperature. This result, which is not in conflict with the second law, does not preclude obtaining results for other models that are in conflict and all that can be asserted on the basis of these results is that for this diode model there is no conflict.

The direction of the rectified current in the diode models can be changed by making T_e larger than T_r when $V = 0$ as can be seen from Eqs. (17) and (19). This result is also in agreement with the experimental result reported by Gunn[14] where the direction of the rectified current through an unbiased diode was changed by raising or lowering the temperature of a resistor connected across the terminals of the diode. For this experiment the current was observed to flow in the reverse (high-resistance) direction through the diode when the resistor was at a lower temperature than the diode and this is the same result as would be predicted from Eq. (17) for the diode R_e in Fig. 1.

POWER CONVERSION EFFICIENCY AND REFRIGERATION PERFORMANCE

The power available from or required for this direct current I_t is given by

$$P_w = VI_t = [e(m+n)/C]I_t. \tag{20}$$

The power transferred between the heat reservoirs that generates this direct current output power or that results from a direct current input power is given by

$$P_h = \sum_N P_h(N) = \sum_N \frac{e(N - \frac{1}{2} + n)}{C} I(N), \tag{21}$$

where $P_h(N)$ is the power transferred between reservoirs for each N. Then the efficiency of the power conversion mode is given by

$$E = P_w/P_h. \tag{22}$$

In addition to the reversal of current flow that results when T_e becomes larger than T_r for $V=0$, Eqs. (17) and (19) also show that a reversal of current flow will occur as the voltage V is increased to a larger value than the expected fluctuation voltage. Then the model no longer converts the thermal fluctuation power P_h from the higher-temperature diode R_r to output power P_w. Above the voltage at which the current direction is reversed the power P_w becomes the input power to the model and the power P_h becomes the power transferred to the higher temperature diode R_r. When this transferred power P_h is larger than the input power P_w, then the thermodynamic cycle is operating in a refrigeration mode. For this mode, the output is the heat extracted from the cold reservoir $P_h - P_w$ and the input power is P_w. A convenient measure of performance of the refrigeration mode is the coefficient of performance ω which is the ratio of the above quantities or in terms of E is given by

$$\omega = (P_h - P_w)/P_w = (1-E)/E. \tag{23}$$

It can be seen from this that when the input power becomes larger than the heat transferred to the hot reservoir to make $E > 1$, the coefficient of performance of the refrigeration mode goes to zero.

It is of interest to determine the highest efficiency obtainable from this thermal engine and this would be expected to occur when the thermal fluctuations of small systems were being ... the limit as the capacity C decreases and ... thereby increase, then the number of terms ... tributing to P_w and P_h approaches one. This can be seen from the equation for the probability distribution given by Eq. (11) and the equation for the rectified current given by Eqs. (17) and (12). For this example and for the other models that were analyzed, it can be shown that as the factor $(\beta - \alpha)$ in Eq. (17) increases, the fraction of the rectified current at the value of N at which $I(N)$ becomes positive also increases. This result can be understood by noting that while $I(N)/P(N)$ continues to increase rapidly for larger values of N, the value of $P(N)$ decreases at a faster rate so that in the limit all the rectified current occurs at the value of N for which the value of $I(N)$ becomes positive. For this limiting case, the efficiency E_0 approaches the ratio

$$E_0 = (m+n)/(N_0 - \tfrac{1}{2} + n), \tag{24}$$

where N_0 is the value of N for which the rectified current occurs in this limiting case.

Then to examine the value of E_0 at the point at which the reversal of current occurs, we have from Eq. (17) or (19)

$$\beta m + \alpha n = (\beta - \alpha)(N_0 - \tfrac{1}{2})$$

and

$$m + n = [(T_r - T_e)/T_r](N_0 - \tfrac{1}{2} + n). \tag{25}$$

Substituting into Eq. (24) gives

$$E_0 = (T_r - T_e)/T_r. \tag{26}$$

This is the Carnot efficiency so again the thermal engine for this model is in agreement with the second law. This result for the efficiency in this limiting case holds for all series and parallel combinations of diodes that were analyzed. This result for models using Alkemade diodes indicates that the efficiency of the power output is limited to that of the Carnot cycle.

It is of interest to compare the above results using diodes for R_r and R_e with the results for a model where R_r is a linear resistor. The results for a preliminary analysis are, as expected, that similar efficiencies can be achieved for this model. For this analysis a continuous fluctuation voltage distribution must be used. Let us assume, as for linear resistors, we can represent the voltage distribution V_t across the C in Fig. 1 by the equation

$$P(V_t) = P(V_m)\exp\left(-\frac{(V_t - V_m)^2}{2(\sigma_r^2 + \sigma_e^2)}\right), \tag{27}$$

where V_m is the bias voltage across C and where σ_r^2 and σ_e^2, the variances of the voltage fluctuations of R_r and R_c, respectively, are given by

$$\sigma_r^2 = (kT_r/C)R_p/R_r, \quad (28)$$

$$\sigma_c^2 = (kT_c/C)R_p/R_c, \quad (29)$$

with R_p, the parallel resistance of R_r and R_c across C, being given by

$$R_p = (1/R_r + 1/R_c)^{-1}. \quad (30)$$

For the case of interest to us, where either or both R_r and R_c are nonlinear, the voltage fluctuation across C is not normally distributed and the use of the above distribution for this model is open to question. To give some plausibility to this assumption, the above voltage distribution does give a Maxwell-Boltzmann distribution for the energy being generated and dissipated by R_r and R_c in the model. Also the model does give the correct distribution for the limiting case, where R_c becomes an ideal rectifier as $T_c \to 0$. For this limiting case, $R_c \to 0$ when the diode conducts so that the voltage across C is limited to the value of V_t at which the diode starts to conduct.

On the basis of this assumed voltage distribution and the equation for the expected rectified current given by Eq. (16), where $N = V_tC/e$, the value of P_e, P_b and E can be computed using Eq. (20), Eq. (21), and Eq. (22), respectively. For the simplifying assumption that $\beta \gg 1$, it can be easily shown that the efficiency E is given by the Carnot cycle efficiency $(T_r - T_c)/T_r$ as in Eq. (26) for the diode model.

The similar efficiency for the above models can be shown to be examples of a more general model for the rectification of energy fluctuations. This general model for the conversion of electric energy fluctuations includes a nonlinear resistive circuit element maintained at temperature T_c and a heated resistive element maintained at temperature T_r. Assume the electric fluctuations of the coupled elements result in the following currents through the nonlinear circuit element at temperature T_c. The current flow in the forward direction is

$$I_f = A \exp[(\Delta E - \Delta W)/kT_c] \quad (31)$$

and in the backward direction is

$$I_b = A \exp[\Delta E/kT_r], \quad (32)$$

where ΔE is the change in the fluctuation energy of the model for a current flow of one electron or ion through the nonlinear resistor as, for example, given by Eq. (1) and ΔW is the output energy of the circuit for this current flow. Then for current flow at values of ΔE for which the output energy ΔW of the model from this charge flow approaches $\Delta E(T_r - T_c)/T_r$, it can be seen that the efficiency of the power conversion approaches the Carnot efficiency. Also, it can be shown in general, for either the fluctuation energy distribution of all the Alkemade diode models, or the continuous voltage fluctuation distribution of the models with heated linear resistors, that decreasing the temperature T_c of the nonlinear circuit element R_c or the capacity C of the system will in the limit allow all power conversion to occur at the value of ΔE for which the Carnot efficiency is achieved.

The fact that this result based on statistical mechanics is in agreement with thermodynamics is an interesting result. It was suggested in the review that the energy of dissipative fluctuation processes (Brownian motion in a general sense) may be fundamentally synonymous with heat. Without answering this question, the results for all the models that were considered do add to the reasons for considering this question further.

DISCUSSION OF RESULTS

Although it is evident from Eq. (26) that this cycle is a reversible cycle, it is more difficult to accept the concept of a diode working in a refrigeration mode than in a power conversion mode. A limited discussion by Feynman of the ratchet and pawl machine[20] is useful to the understanding of the sometimes surprising results of reversible cycles. In this discussion he assumes a reversible cycle and then discusses the physics of the ratchet as an engine for each mode. The fact that the concept of a diode working in a refrigeration mode is a surprising result is only one more example of a conclusion reached in a study by Keenan *et al.*[21] on the fuel shortage and thermodynamics where it is shown that a general understanding and attention to the thermodynamics of reversible processes and a reinvigorated attack on irreversible processes can have a great potential in saving fuel.

Examples have been computed to show the relation between the parameters in Eq. (22). For simplicity in the computation for all of the examples, it is assumed that $r = K$ where r is the ratio m/n. This assumption corresponds to the physical model where the capacity, work functions, and saturation currents per unit surface area of each diode are the same so that the ratio of the saturation currents of the two diodes K would be equal to the ratio of the capacities of the two diodes. And, since the ratio of diode capacities determines the ratio of direct voltages from the battery across each diode, it follows that $r = K$ for this model. Other physical models can, of course, change this relation in either direction.

Using this model and the values of r and K, given in Table I, the values of $P(V)$ from Eq. (11), $I(V)$

from Eq. (17) and $P_a(N)$ from Eq. (21) are given in this table for $N=-3$ to $N=3$ for two values of m. The lower value of m gives a positive value of I_t and a power conversion efficiency E from Eq. (22) of 83%. The higher value of m gives a negative current for I_t and a negative power output for P_w. From this example, it can be seen that with large values for α and β, effectively all the direct current occurs for $N=1$.

The relation between m and E for another example is shown in Fig. 2 for a wider range of m. For this example, it can be seen that the increase in E is almost linear up to the cross over point after which the value of E continues to increase at approximately the same rate. The maximum efficiency of power output for this example is 64% and the minimum value of E for the refrigeration mode after the cross over point is 78%. This minimum value of 78% for E gives the maximum value of the performance coefficient ω of 28%.

The results of the computation of the conversion efficiency as a function of the parameters of the model show the expected agreement with the basic concepts of fluctuation theory. These results showed that the conversion efficiency increases as the system size decreases to make the fluctuation energy a significant fraction of the total energy. The decrease in system size is required to increase the values for α and β.

These results that there are no theoretical limitations for this model in approaching the efficiency limitations of the Carnot cycle are based on the assumption that electric fluctuations are the only energy exchanged between diodes. These assumptions cannot be rigorously met in any physical model, however, it is not evident that there are any physical limitations to designing a model capable of meeting the assumptions as close as is required to make the above theoretical performance equations essentially correct. The basis for this conclusion with respect to the above assumption is that since the conversion model involves a rectification of electric energy fluctuations, it is feasible to have as a design goal the reduction to a negligible amount the losses due to thermal conduction, radiation and electron cooling. In theory, filters and conductors can be designed that will convey electric power efficiently over a selected wide part of the electric fluctuation frequency spectrum and at the same time provide poor thermal conduction paths. Designs to do this are discussed in the Appendix.

TABLE I. Power conversion and refrigeration mode.

		$\beta=75$	$\alpha=7.5$	$K=r=10^3$
Power-conversion mode:		$I_t=4.25$	$P_a=2.16$	$m=0.42$
Refrigeration mode:		$I_t=-3.86$	$P_a=-0.178$	$m=0.46$
N	m	$P(N)$	$I(N)$	$P_a(N)$
−3	0.42	0.213×10^{-14}	-6.32×10^{-27}	$2.21\times10^{-[illegible]}$
	0.46	0.212×10^{-14}	-6.32×10^{-27}	$2.21\times10^{-[illegible]}$
−2	0.42	0.296×10^{-4}	-1.58×10^{-15}	3.96×10^{-15}
	0.46	0.294×10^{-4}	-1.58×10^{-15}	3.96×10^{-15}
−1	0.42	0.226×10^{-1}	-2.19×10^{-7}	3.29×10^{-7}
	0.46	0.225×10^{-1}	-2.19×10^{-7}	3.29×10^{-7}
0	0.42	9.962	-1.68×10^{-2}	8.41×10^{-4}
	0.46	0.954	-1.68×10^{-3}	8.40×10^{-3}
1	0.42	0.170×10^{-1}	4.29	2.15
	0.46	0.230×10^{-1}	−0.369	−0.184
2	0.42	1.13×10^{-17}	1.61×10^{-4}	2.41×10^{-4}
	0.46	0.308×10^{-15}	2.21×10^{-4}	3.31×10^{-4}
3	0.42	$0.201\times10^{-[illegible]}$	5.99×10^{-43}	1.50×10^{-41}
	0.46	$1.10\times10^{-[illegible]}$	1.65×10^{-41}	4.13×10^{-41}

POWER-OUTPUT POTENTIAL

To determine if there is any practical significance to the above theoretical limits, it is necessary to investigate the feasibility of using assemblages of microscopic circuits of the above model. One result of a preliminary examination of the theoretical limit on the available fluctuation power per square meter of a heated resistive film is that this limit is extremely high. In computing the magnitude of the upper limit on the available fluctuation power, let us first consider the fluctuation power of a single resistor. The effect of the statistics of fluctuation effects on the thermally caused electrical fluctuations across a single resistor is that the output power is independent of the physical size or the number of conducting electrons in the resistor. This power has been shown

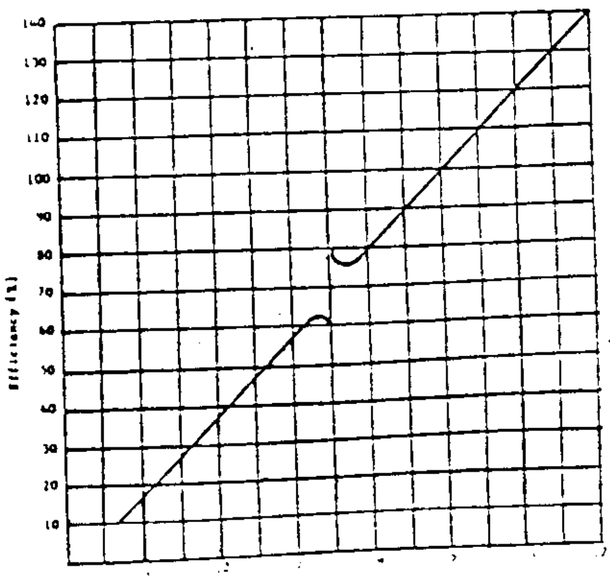

FIG. 2. Efficiency of power conversion and refrigeration mode as function of m for $\beta=29$, $\alpha=8.7$, $K=r=10^3$.

to be approximately kT/t, with k being the Boltzmann constant, T being the absolute temperature, and t being the mean time between collisions for an electron. The effective mean time t to give the fluctuation power in a metal resistor has been computed[22] and the result shows an effective electron velocity of 10^8 cm/sec and an effective mean-free-path length of 10^{-6} cm so that the effective mean time t is of the order of 10^{-14} sec for electrons at room temperature. For this value for t and for $T = 700$ °K the fluctuation power available is of the order of 10^{-6} W.

The problem remains of trying to determine the limits in size for a physical system to operate efficiently. For this, let us consider how closely contacts on a heated resistive film can be placed. On the basis that the mean free path of the electrons in the resistive material is of the order of 10^{-6} cm, an order of magnitude estimate is that conductors can be placed every 10^{-5} cm on an extended heated resistive film without resulting in any significant correlation between the fluctuations of the individual circuits. This spacing results in an available theoretical power output limitation of 10^4 W of available fluctuation energy per square meter of a resistive film at a temperature of 700 °K.

A model to illustrate the physical size requirements in three dimensions of the fluctuation power conversion circuit is shown in Fig. 3. The thermal wall separating the two temperature regions T_r and T_c is in the XZ plane between or congruent with the plates of the capacitors C_1, C_2, and C_3. As will be discussed in the Appendix, these capacitors can transmit the electrical fluctuation energy without relaying unwanted thermal energy. The use of the two diodes in the R_c circuit is one model

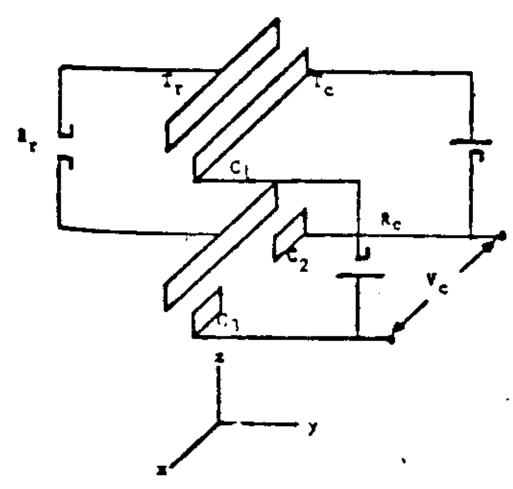

FIG. 3. Power conversion circuit for unit cubicle cell in assemblage.

to enable a rectification current to exist in the lower-temperature region without requiring a dc path to the higher-temperature region. The diode R_r in the higher-temperature region is assumed for simplicity to be one with a cathode and anode having the same work function so that the fluctuation probability distribution of the fluctuation current between the electrodes is easily seen to be without any bias.

The power output is taken from terminals at the voltage V_c as indicated in Fig. 3. The computation of the power conversion efficiency for this model also results in the Carnot efficiency being achieved for the small circuit size. A similar efficiency can be shown to result if a linear resistor or resistive film was used in place of R_r in the model.

Then to consider the minimum volume required for this circuit, it is evident that if an assemblage of these circuits were placed in cubical cells in the XZ plane, the thickness of this assemblage would be comparable to the spacing of the circuits in the XZ plane. The above assumed spacing of 10^{-5} cm in this plane is a possible goal since recent work in electron beam microfabrication[16-19] has demonstrated that linewidth of less than 10^{-5} cm can be obtained, and progress is expected towards still smaller linewidths. To give some purpose to this development for the long term future, the ability to convert 10^{-6} W of power in a cube 10^{-5} cm on a side means that with only a total volume of 1 m³ of these small circuits, there is a total potential power conversion output sufficient to provide each individual in a world population of ten billion with 100 kW of electrical power. As another comparison of this theoretical potential, this power output is equal to 1% of the solar energy intercepted by our planet.

These results indicate that the major problem is not the available power, but rather the efficiency with which this available power can be converted. This problem area involves a material and fabrication study designed to make maximum use of the promising possibilities for achieving high efficiencies. If the material and fabrication investigation did make possible the achievement of these advantages then this technique could yield practical amounts of power at high efficiencies over very wide ranges of temperatures.

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APPENDIX

The analysis of the ideal efficiency of the direct conversion of fluctuation energy showed that, for this concept, it was in theory possible to design models for which there was no significant heat loss by thermal conduction or radiation from the hot to the cold reservoirs. As a first example, let us consider how a model with the same master equation as that for the model of Fig. 1 can be designed with a minimum heat transfer loss between heat reservoirs by eliminating the dc path between R_r and R_c. For this design the fluctuation energy spectrum can be transmitted between R_r and R_c by capacitors C_1 and C_2 as shown in the Fig. 4. In this circuit the inductances L_r and L_c are added to provide a dc return path for the rectified current through each diode so that no dc path is required between heat reservoirs. The impedance of the inductances is assumed to be much higher at the frequencies of the fluctuation voltage than the impedance of the diodes so as to make the dc current the only significant current through the inductances. To enable the master equation given in Eq. (6) for the circuit of Fig. 1 to also be the master equation for the circuit of Fig. 4, the voltages V_r and V_c are set to give the same voltage across each diode as V, the sum of the voltages V_1 and V_2, gives for the circuit of Fig. 1. This will make the dc currents, I_r and I_c, to be equal so that the resulting conversion power output and efficiency will then be the same as for the circuit of Fig. 1.

As another example, let us consider a model using only diodes to provide the dc circuits for each temperature region as shown in Fig. 5 with the fluctuation energy being transmitted between the temperature regions T_r and T_c by capacitors C_1, C_2, and C_3. Rectification and power conversion can occur in either or both of the circuits R_r and R_c. The power conversion can be derived using the master equation for the electron emission probability distribution as was done for the model in Fig. 1. Using the same notation, the recursion equation for the equilibrium state is

$$P(N)=\frac{K\exp[-\alpha(N-\frac{1}{2}+n)]+K+\exp[-\beta(N-\frac{1}{2}+m)]+1}{K\exp[\alpha(N-\frac{1}{2}-n)]+K+\exp[\beta(N-\frac{1}{2}-m)]+1}\,P(N-1), \tag{33}$$

where $K=J/A$, $V_r=2ne/C$, and $V_c=2me/C$. The equations for the dc currents I_r and I_c can then be derived using

$$I_r=\sum_N I_r(N), \tag{34}$$

where

$$I_r(N)=J\{\exp[\alpha(N-\tfrac{1}{2}-n)]-1\}\times P(N)-J\{1-\exp[-\alpha(N-\tfrac{1}{2}+n)]\}P(N-1) \tag{35}$$

and

$$I_c=\sum_N I_c(N), \tag{36}$$

where

$$I_c(N)=A\{\exp[\beta(N-\tfrac{1}{2}-m)]-1\}\times P(N)-A\{1-\exp[-\beta(N-\tfrac{1}{2}+m)]\}P(N-1). \tag{37}$$

From these currents the power output P_w can be obtained and is given by

$$P_w=(e/C)(mI_c+nI_r). \tag{38}$$

Also the power into the circuit R_c is given by

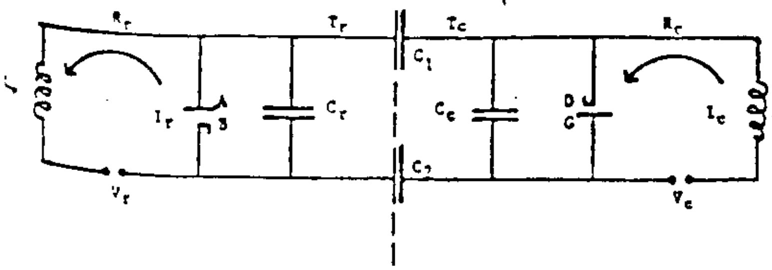

FIG. 4. Circuit for power conversion to minimize heat-conduction losses.