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## Creation Model of the Bostick Plasmoid in Magnetic-Field-Free Space

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A creation model has been developed to interpret the behavior of Bostick plasmoids in the plane of the discharge. The plasmoids are formed by pulsed unidirectional current discharges across the electrodes of a button gun in vacuo  $(10^{-6} \text{ Torr})$ . The model is that of a vacuum arc which expands in the form of a thin, highly conducting torus by virtue of the interaction of the surge current with its self-induced magnetic field. When the current no longer flows, the plasma of the arc continues to move by virtue of its own inertia. The plasmoid speeds directly away from the button gun have been determined experimentally from time of arrival measurements to moveable electrostatic probes, and are found to increase monotonically from 20 to 100 km/sec for a corresponding source voltage range of 2.5-17.5 kV. The measured velocity v' of any point on the plasma which subtends an angle  $\theta$  to the direction of forward motion at the button-gun source is given by  $v' = \langle I \rangle (\alpha/\pi\rho)^{1/2} \cos\theta$ , where  $\langle I \rangle$  is the average surge current,  $\rho$  is the measured mass per unit length (line density) of the genetic plasmoid and  $\alpha$  is a theoretically calculated constant. The line density  $\rho$  of the plasmoid has been shown theoretically to be a constant and its magnitude has been measured to be  $10^{-9}$  kg/m and hence the mass M of the plasmoid is deduced theoretically to be given by:  $M = \langle I \rangle \tau (\alpha \pi \rho)^{1/2}$ , where  $\tau$  is the duration of the current pulse. This formula predicts the plasmoid mass to increase from 0.05 to 0.2  $\mu$ g for the current range of interest and the predicted mass is verified to within 16% by a novel experimental technique.

## I. INTRODUCTION

Over twelve years have elapsed since Bostick first discovered that a high-current capacitor discharge, pulsed across the electrodes of a button gun *in vacuo*  $(10^{-5}$  Torr) resulted in the ionized material of the vacuum arc being projected from the button gun at speeds of up to 200 km/sec.<sup>1</sup> The early studies of these bursts of rapidly moving plasma consisted of measuring their forward velocity as a function of the parameters of the experiment.<sup>2</sup> These velocities were deduced from a time of arrival method by using electrostatic probes to measure the polarization field induced in the plasma as it traveled through dc magnetic fields of various intensities. The number of ions in an arbitrary plasmoid was estimated to lie between  $10^{15}$  and  $10^{18}$  for a corresponding current range of 1000-10 000 A.<sup>1</sup>

On the basis of evidence deduced from an analysis of oscillographic signals received from the electrostatic probe, and from Kerr cell photographs of the plasmoid in a higher ambient pressure  $(10^{-3} \text{ Torr})$  than that in which probing experiments were performed, the plasmoid was believed to move through space as a highly conducting torus around which a current flowed by virtue of the flux trapped within the torus.<sup>1</sup> The word "plasmoid" was originally chosen to describe this plasma magnetic entity, and it was believed that the plasma retained this toroidal structure even in the absence of externally induced magnetic fields.

It should be noted, however, that very little real information is available about the fundamental physics of plasmoids. For example, the toroidal geometry which the plasma was believed to assume during its formation period was proposed more as an intuitively obvious concept rather than as a deduction from experimental fact. A torus has never been successfully photographed in the high vacuum in which probing experiments were performed. The reason is clear—in a high vacuum, the plasmoid is moving too rapidly and giving off too little recombination light to be recorded photographically.

A significant advance toward an understanding of plasmoids was achieved when it was discovered experimentally that the forward speeds of plasmoids depended only upon the energy of the source capacitor and hence upon the surge current which was pulsed through the button-gun circuit.<sup>2</sup> Although the dependence of plasmoid velocity on surge current was clearly recognized, no attempt seems to have been made to explain or interpret this result, and the discovery itself appears to have attracted little attention in the literature. Recently, however, a theoretical analysis of Bostick's pioneering work revealed that the assumption of a highly conducting toroidal plasmoid necessarily implied that the forward velocity of the plasma was proportional to the mean surge current, and it was also shown that the number of ions in an arbitrary plasmoid could be estimated theoretically in terms of the parameters of the experiment.<sup>3</sup> Later, it was shown theoretically by a simple argument that a plasmoid was not likely to be projected as an isolated torus in the manner hypothesized by Bostick,<sup>1</sup> but that if the genetic structure was toroidal, it was necessary that the motion of the plasma front in the plane of the discharge be simulated by a circle which expanded with one point fixed at the button source, and direct measurement correlated well with the predicted profile.<sup>4</sup> Recently, it was verified experimentally that the forward velocity of plasmoids was directly proportional to the mean surge current over a large range of source voltage.5

The information which has been accumulated about plasmoids up to the present time, seems to indicate



FIG. 1. The discharge circuit.

strongly that considerable insight into the dynamics of plasmoids might be obtained if attention was focused upon their creation mechanism because it was believed that in the absence of externally induced magnetic fields, once the surge current had ceased to flow, the inertia of the moving plasma would enable its behavior to be predicted in terms of the experimental parameters. However, any attempt to systematically analyze the creation dynamics of plasmoids would seem to require rather more information than is presently available in the literature, and it is for this reason that an experimental project was undertaken to obtain sufficient data to enable an attempt to be made to answer the following questions.

(1) Is it possible to construct a model of a plasmoid which will simulate the observed motion in the plane of discharge?

(2) What is the mechanism of detachment of a plasmoid from its button source?

(3) What experimental parameters control the physical properties of a plasmoid?

(4) How does the mass of a plasmoid depend upon the parameters of the experiment?

The contents of this paper describe an experimental and theoretical investigation which was undertaken with a view to answering these questions. The investigation is concerned with one particular aspect of plasmoid physics which has not been discussed in the literaturenamely the measurement and interpretation of the motion of plasmoids in the plane of the current discharge, and it will be shown how previous work may be correlated with the concepts which this paper introduces (a) to describe a self-consistent model of a plasmoid and (b) to show that the entire history of a plasmoid is determined by, and may be predicted from its creation dynamics. The apparatus and experimental techniques, together with the results which are obtained, will be described in detail, and the subsequent analysis reveals that a toroidal model of a plasmoid is an explicit consequence of the experimental results. This model will be shown to be capable of answering questions (1)-(4) above. Finally, the contents of the paper will be summarized and discussed, and it will be

shown that the present work is not at variance with that reported in the literature.

## **II. APPARATUS**

It already has been stated that the genetic structure of a plasmoid is still a matter of pure speculation because no photographs as yet exist of the plasmoid during its early creation period in the high vacuum in which probing experiments were made. Therefore, since the geometry of the plasmoid is unknown, it is not possible to perform a rigorous analysis of its creation dynamics. Nevertheless, it might have been possible to gain considerable insight into plasmoid physics if highly dependable measurements of various aspects of its behavior existed-such as, for example, velocity measurements. However, the attempts which were made by the pioneering workers in the field appear to have been only partially successful because of the lack of reproducibility of signals obtained from electrostatic probes which were placed in the path of the plasmoids. This lack of reproducibility was attributed to a lack of understanding of the behavior of the probes,<sup>2</sup> and this is almost certain to be one of the main difficulties which has delayed the hypothesis of a model which will satisfactorily interpret the behavior of plasmoids. It is because of this lack of reliable data that a different method of measuring plasmoid speeds has been constructed and is described below.

The essential features of the apparatus are shown in Fig. 1. The plasmoids are obtained by switching a 0.18- $\mu$ F capacitor across the electrodes of a conventional button gun by a standard three electrode spark gap switch operated in air. Nonlinear damping resistors<sup>6</sup> were used to shape the discharge current to ensure that each capacitor discharge produced a single unidirectional current pulse. The plasmoids which were formed by these capacitor discharges were blown from the button gun and their times of arrival to a moveable electrostatic probe placed directly in their path were recorded oscillographically. This probe consisted of two



FIG. 2. Forward velocity of plasmoid vs source voltage.

short (1 cm) lengths of 1-mm-diam copper wire mounted parallel to each other. The separation of the electrodes was 3 mm. The electrodes of the probe were connected in series with a 225-V battery and a 10-k $\Omega$ resistor. Thus, when the conducting plasmoid passes between the electrodes of the probe the region between the electrodes temporarily conducts and the resulting potential drop across the 10-k $\Omega$  resistor was recorded on a cathode-ray oscilloscope. The oscilloscope was triggered at the instant of initiation of the surge current in the circuit of the button gun by the output from standard Rogowski belts.<sup>7</sup> Thus, time of arrival measurements of the plasmoid were made and hence the velocity of the plasmoid in the vacuum of 10<sup>-6</sup> Torr could be deduced.

#### **III. EXPERIMENTAL RESULTS**

The preliminary experiments which were performed are concerned with measuring the forward velocity of the plasmoids as a function of source voltage. The reason for choosing this particular aspect of plasmoid physics as the subject for initial study was to attempt to correlate the present measurements with those of past workers who placed emphasis on the behavior of plasmoids moving in this direction. In the present work, the forward velocities of the plasmoids were determined in the following manner: The electrostatic probe was located directly in front of the button gun and 5 cm from it, and times of arrival were measured up to distances of 40 cm in steps of 5 cm. Each determination of arrival time was taken as the mean of four measurements because it was found that the scatter of results was such that the average of any four measurements of arrival time differed from any one of the measurements by somewhat less than 10%. In this way, the forward velocity of the plasmoids was measured as a function of the source voltage for voltages of 2.5, 3.75, 5.0, 6.25, 7.5, 10.0, 12.5, 15.0, and 17.5 kV. It was found that this velocity increased monotonically with source voltage throughout the entire range of interest (see Fig. 2). It is clear that the shape of our velocity vs source voltage curve does not correlate even approximately with that obtained by similar experiments performed by the pioneer workers although the velocities which are measured are of the same order of magnitude as those which have been reported in the literature.<sup>2</sup> The remainder of this paper will be taken up with an attempt to interpret the results in Fig. 2.

#### **IV. THEORETICAL INTERPRETATION**

An exact analysis of plasmoid dynamics from the instant of plasmoid formation is impossible because an exact knowledge of the geometry of plasmoids is presently lacking. It should also be noted that although the assumed toroidal model is what could be expected on intuitive grounds, such a structure has no real experi-

mental foundation, but was originally proposed by Bostick to explain qualitatively the shape of oscillographic signals which were received from electrostatic probes.<sup>1</sup> A toroidal structure has, however, been photographed when the plasmoids were created in a high ambient gas pressure (10<sup>-3</sup> Torr),<sup>1</sup> but this pressure is considerably higher than the pressure at which probing experiments were performed  $(10^{-5} \text{ Torr})$ , and it was from probe signals in this higher vacuum that the original model was proposed. Therefore, it is not clear that the plasmoid would not change its fundamental properties with such a change in pressure. Accordingly, the procedure which will be adopted in this paper, will be a consolidation of the results which are recorded in the relevant literature to construct a physically reasonable and self-consistent model which will simulate the behavior of "real" plasmoids. Our model depends on three a priori assumptions whose necessity is implicit in the present lack of knowledge of the genetic structure of an arbitrary plasmoid. These assumptions, however, will be clearly stated, and later verified by corroborating experiments. We shall start by assuming a genetic geometry for the plasmoid, and, by utilizing simple energy equations, we shall deduce equations of motion whose implications and ramifications will be explored theoretically and tested experimentally to reveal a self-consistent model of the Bostick plasmoid.

In view of the fact that certain experimental and theoretical work on plasmoids is strongly suggestive of a toroidal geometry,<sup>3,4</sup> we shall assume that such is indeed the characteristic geometry and we make our first assumption.

Assumption 1: During its creation period, the Bostick plasmoid grows as a torus with respect to its center of mass, and the torus expands by virtue of the interaction of the surge current with its self-induced magnetic field and maintains its geometry by the pinch fields at its surface.

At this point we emphasize that if the geometry of the plasmoid is indeed toroidal, then it is necessary that the motion be simulated by a uniformly expanding torus which has one point fixed at the button gun. The physical reason for this is easy to understand, because if the genetic geometry of the plasmoid is indeed toroidal, then it is necessary that the velocity of any point on the periphery relative to the center of mass equals the velocity of the center of mass relative to the button gun. It follows that at the bitton gun, there is no resultant motion of the plasma relative to the button gun. This argument has been given elsewhere and verified experimentally for one source voltage.<sup>4</sup>

We continue our investigation by consideration of the energy which is injected into a given plasmoid by virtue of a capacitor discharge. It has been shown elsewhere that an assumption of high conductivity en-

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FIG. 3. Typical I-V transients. Voltage trace taken across coaxial cable. Sweep:  $0.5 \,\mu$ sec/cm. Current sensitivity:  $1.5 \,\text{kA/cm}$ . Voltage sensitivity:  $2.5 \,\text{kV/cm}$ . Source voltage:  $10 \,\text{kV}$ . Traces going from right to left.

ables sufficient insight to be obtained into plasmoid dynamics to allow some correlation to be made between various properties of plasmoids.<sup>3</sup> A simple experimental test of this assumption would consist of simultaneously recording the voltage across the button gun and the corresponding surge current. However, attempts to measure the voltage directly across the button-gun electrodes were unsuccessful because of large amounts of electromagnetic noise which completely obscured the oscillographic transient. Accordingly, the voltage was measured across a short (1 ft) strip of coaxial cable which successfully choked out the undesirable high frequencies so that clean photographs of simultaneously recorded current and voltage transients could be obtained (see Fig. 3). It is clear that these transients are almost exactly 90° out of phase with each other, thus showing that the Ohmic resistance of the plasmoid is negligible. Hence, we make our second assumption.

Assumption 2: During the time that the plasmoid is being created by the button gun, Ohmic losses may be neglected.

It was found by a numerical integration of transients such as those shown in Fig. 3, that there was always a small but definite energy loss which disappeared when the button gun was shorted out. Other energy losses such as electrode evaporation due to heating, ionization energies, and so on were estimated and found to be negligible in comparison. [An interesting phenomenon is revealed by this study. See: D. A. Butter, Phys. Fluids **13**, 770 (1970).] We therefore interpret this energy loss as the energy which is imparted to the plasmoid by the capacitor discharge, and, since the Ohmic losses are small, it follows that the magnetic energy of the plasmoid is converted into kinetic energy of motion. Hence, using our second assumption, we write the magnetic energy W of the plasmoid at any instant during its formation:

$$W = \frac{1}{2}LI^2,\tag{1}$$

where L is the self-inductance of the plasmoid and I is the surge current at that instant. In principle it is easy to find the radially directed force  $F_r$  from

$$F_r = \partial W / \partial r, \qquad (2)$$

where r is the major radius of our assumed torus. Unfortunately, this approach presupposes a knowledge of  $\partial I/\partial r$  which is not available because of our lack of knowledge of how the radius r behaves as a function of time during its early stages of formation. At this point, therefore, we make our third assumption.

Assumption 3: The motion and behavior of the genetic plasmoid during its formation period while the true time-dependent current I is flowing, may be simulated by the flow of a rectangular current pulse  $\langle I \rangle$ , whose duration is the same as that of I and whose magnitude is that of the average value of I.

We may use our third assumption in conjunction with Eqs. (1) and (2) to deduce

$$F_r = \frac{1}{2} \langle I \rangle^2 (\partial L / \partial r).$$
(3)

Now, the self-inductance L of a torus in mks units is given by<sup>8</sup>

$$L = 1.257 r [\ln(8r/r_0) - 2 + \mu \delta] \times 10^{-6} \text{ H}, \qquad (4)$$

where  $r_0$  is the minor radius of the torus and  $\mu$  and  $\delta$  are parameters which depend, respectively, on the material of the torus and the frequency of the current flow. For a plasma,  $\mu = 1$  and for a constant current  $\langle I \rangle$ ,  $\delta = 0.25$ .<sup>8</sup> We write

$$\alpha(\mathbf{r}) = 1.257 [\ln(8\mathbf{r}/r_0) - 1.75] \times 10^{-6}.$$
 (5)

We now shall investigate the parameter  $\alpha(r)$ . The magnetic energy W in the plasmoid is given by

$$W = \frac{1}{2} \int_0^R I^2 \frac{\partial}{\partial r} \left[ r\alpha(r) \right] dr + \int_0^R r\alpha(r) I \frac{\partial I}{\partial r} dr,$$

where we have assumed that the initial radius of the plasma loop is 0 and the final radius, at the instant of cessation of the surge current as being R. Now, the surge current is zero at both these limits of integration and hence, we use the first theorem of mean value<sup>9</sup> to deduce that

$$W = \frac{1}{2} R \alpha(R) \langle I \rangle^2, \qquad (6)$$

where  $\langle I \rangle$  is the magnitude of the surge current when the radius of the plasmoid is some value between 0 and *R*. We shall interpret  $\langle I \rangle$  as being the mean current. Now, the duration of the surge current is 1  $\mu$ sec (see Fig. 3) and hence the value of *R* for different surge currents was measured by moving the electro-

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static probe along the direction of the forward trajectory of the plasmoid until a signal was registered 1  $\mu$ sec after initiation of the surge current. The distance between the probe and the button gun was then taken to be the diameter of the assumed torus. It was found that  $\alpha(R)$  was a constant for the entire range of source voltage and the ratio of the major to minor radii was, therefore, also a constant, deduced to be about 100. In other words, if our toroidal model is correct, it is necessary that the torus be thin, and we deduce from Eq. (3) that  $F_r$  is a constant, given by

$$F_r = \frac{1}{2} \alpha \langle I \rangle^2. \tag{7}$$

Let us now define  $\rho$ , the mass per unit length of our assumed torus, so that we may write Newton's second law in the center of mass coordinates:

$$\frac{1}{2}\alpha \langle I \rangle^2 = (d/dt) \left(2\pi\rho r\dot{r}\right). \tag{8}$$

It has been shown elsewhere that  $\alpha$  is directly proportional to  $\rho^3$  and, since  $\alpha$  has been shown experimentally to be a constant it follows that  $\rho$  is a constant also, and hence, we deduce from Eq. (8) that the equation of motion of the plasmoid periphery relative to an observer in the center of mass coordinates is given by

$$\left(\frac{d^2}{dt^2}\right)\left(r^2\right) = \alpha \left\langle I \right\rangle^2 / 2\pi\rho.$$
(9)

We choose our boundary conditions to be r=0, and  $\dot{r}$  is finite at t=0 and hence deduce the solution of Eq. (9) to be

$$\dot{r} = \frac{1}{2} \langle I \rangle (\alpha / \pi \rho)^{1/2}.$$
 (10)

We have, therefore, found an expression for the velocity of the plasmoid periphery relative to an observer in the center of mass coordinates. It now is necessary to deduce an expression for the forward velocity of the plasmoid relative to an observer in the laboratory frame of reference which may be done simply by the following argument: It is clear from Eq. (10) that an observer situated on the button gun (i.e., situated in the laboratory coordinates) will see the center of mass of the plasmoid moving at the speed  $\dot{r}$ 



FIG. 4. Peak and mean currents vs source voltage.



FIG. 5. Forward velocity of plasmoid vs mean current.

and hence the velocity v (say) of the front edge of the plasmoid will be measured to be  $v=2\dot{r}$ , or

$$v = \langle I \rangle (\alpha / \pi \rho)^{1/2}.$$
 (11)

Now, the mean current  $\langle I \rangle$  is not a linear function of the source voltage, but it can, nevertheless, be computed numerically and its functional dependence on source voltage may easily be found to an accuracy of about 10% (see Fig. 4). We now plot measured plasmoid velocities (see Fig. 1) as a function of the mean current  $\langle I \rangle$  with the help of Fig. 4, and we see that the measured velocity correlates well with the predicted linear dependence on  $\langle I \rangle$  (see Fig. 5). We note that Eq. (11) which was deduced from first principles, closely resembles a similar equation which was derived by a totally different method elsewhere.<sup>3</sup>

We have shown experimentally that the forward velocity of plasmoids is proportional to  $\langle I \rangle$ , and we have also shown experimentally that  $\alpha$  is a constant. It follows from Eq. (11), therefore, that the mass per unit length of our plasmoids is also constant and by rewriting Eq. (11) and substituting the values of the known variables, it is easily verified that to within the precision and conception with which these experiments are performed,  $\rho$  is indeed constant and equal to  $10^{-9}$  kg/m to within a standard deviation of 16% (see Table I).

Thus to within the precision and conception of these experiments, we have verified experimentally our second and third assumptions. We shall check our first assumption—namely that of the toroidal geometry of the plasmoid in two ways: (a) by using this assumed geometry to predict the plasmoid mass as a function of  $\langle I \rangle$  and (b) by showing by direct measurement that the plasmoid profile is circular.

#### A. Plasmoid Mass

The mass M of our plasmoids may be predicted theoretically in a simple manner, for, by definition of  $\rho$ we have

$$M = 2\pi\rho R, \tag{12}$$

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 TABLE I. Computed mass per unit length of plasmoids as

 a function of forward velocity.

Measured plasmoid speed (km/sec)	Computed mass per unit length $(kg/m \times 10^9)$
20	1.0
41	0.9
51	0.8
56	1.0
71	0.8
75	1.1
81	1.3
96	1.0
105	1.1

where R is the radius of the plasmoid at the instant of cessation of the surge current. We use Eq. (10) to find R and substitute this value in Eq. (12) to give

$$M = \langle I \rangle \tau (\alpha \pi \rho)^{1/2}, \qquad (13)$$

where  $\tau$  is the duration of the surge current. Experimental measurement of the plasmoid mass in terms of our toroidal model is equally straightforward, for, it is clear that an observer in the center of mass coordinates will see every point on the plasmoid periphery receding with a velocity  $v = \frac{1}{2} \langle I \rangle (\alpha / \pi \rho)^{1/2}$ . In particular, a point P on the periphery which subtends an angle  $2\theta$ at the center of mass [see Fig. 6(a)] will have, relative to the center of mass, velocity components:  $(v \cos 2\theta)$ ,  $v \sin 2\theta$ ). Since the center of mass is moving with a velocity v relative to the button gun (i.e., relative to the laboratory coordinates), it follows that an observer in the laboratory frame of reference will measure the velocity of P as having corresponding components  $\lceil v(1 + \cos 2\theta), v \sin 2\theta \rceil$ . Again, if relative to the laboratory coordinates the point P has a velocity v' and subtends an angle  $\phi$  to the button gun [see Fig. 6(b)], it follows that the corresponding horizontal and vertical components of the velocity of P is measured to be



FIG. 6. (a) Motion of plasmoid periphery in the center of mass coordinates. (b) Motion of plasmoid periphery in the laboratory coordinates.

 $v' \cos \phi$  and  $v' \sin \phi$ , respectively. Hence, we have

$$v' \cos \phi = v(1 + \cos 2\theta)$$
$$v' \sin \phi = v \sin 2\theta,$$

from which it follows at once that  $\theta = \phi$  and  $v' = 2v \cos\theta$ . Hence, since  $\rho$ , and v are both constant, the kinetic energy E of the torus at any instant when the radius is r, is given by

$$E = 4\rho r v^2 \int_{-\pi/2}^{+\pi/2} \cos^2\theta d\theta$$

and, with the help of Eq. (12) this reduces to

$$E = Mv^2, \tag{14}$$

where M is the plasmoid mass. The physics of this formula is easy to understand: If we imagine a torus



FIG. 7. Comparison of predicted and measured plasmoid mass vs  $\langle I \rangle$ .

with its center of mass fixed and expanding radially, then at any instant when its mass is M and its radial velocity is v its kinetic energy is  $\frac{1}{2}Mv^2$ . If now, at this instant an impulse is imparted to a point on the periphery of the torus radially to that point such that a further velocity v is imparted to the torus, then the kinetic energy of the torus is  $Mv^2$  and its motion simulates that of our model, namely, a torus which expands with one point fixed.

We use our second assumption to deduce that in the absence of Ohmic losses

$$E = \int_{0}^{\tau} VIdt, \qquad (15)$$

where V and I are, respectively, the voltage across and the current through the button gun, and  $\tau$  has the same meaning as in Eq. (13). We use Eqs. (14) and (15) to give

$$M = \frac{1}{v^2} \int_0^\tau VIdt.$$
 (16)



FIG. 8. Comparison of predicted and measured polar profiles of plasmoids in magnetic-field-free space.

Now, the integral on the right-hand side of Eq. (16) may be evaluated numerically from photographs such as those in Fig. 3 to give us an experimental method of evaluating the plasmoid mass. Figure 7 compares and contrasts the experimental determination of the plasmoid mass with that predicted by Eq. (13). It is clear from the figure that the experimental values correlate well with those predicted theoretically. Thus, we have shown that our model is able to suggest a simple and self-consistent method of deducing the mass of the plasmoid as a function of mean current.

## B. Plasmoid Profile in the Plane of the Current Discharge

We again test our first assumption by showing from direct measurement that in the plane of the current discharge, our plasmoids expand as circles. The experimental technique was as follows: The electrostatic probe was located directly in front of the button gun and 10 cm from it and is then moved in the plane of the discharge transversely to the direction of forward motion of the plasmoids and the arrival times noted so that the plasmoid velocity could be deduced as a function of polar angle. Measurements were made for source voltages of 3.75, 5.0, 7.5, and 10.0 kV. The distances that the plasmoids moved in  $1 \mu \text{sec}$  (the duration of the surge current) was then computed and plotted on polar graph paper (see Fig. 8). Circles were then drawn such that their diameters equalled the distance between the button gun and the furthermost plotted point, which in each case was found to lie on the forward trajectory of the plasmoid. It is clear from Fig. 8 that these circles are quite good approximations to the measured profile. This experimental fact is consistent with our deduction from Fig. 6 that the velocity v' of the plasmoid at a point inclined at an angle  $\theta$  to the forward direction is  $v'=2v\cos\theta$ , since this equation is merely the polar equation of a circle. We, thus, have shown that our first assumption not only is able to give a self-consistent method of determining the plasmoid mass in terms of our model, but by direct experiment, it has also been shown that the motion in the plane of the discharge correlates quite well with that predicted by our model.

## V. SUMMARY

In what follows, we shall compare and contrast our experimental results and their interpretation with those obtained by Bostick<sup>1</sup> and by Harris *et al.*<sup>2</sup> Since some of the experimental conditions under which the present investigations were performed differ from those of past workers in the field, the aspects of the present work which are at variance with those of Bostick<sup>1</sup> and Harris *et al.*<sup>2</sup> will be discussed in some detail.

#### **A. Experimental Parameters**

No experimental investigation as yet seems to have been undertaken to measure the effect of electrode composition on the properties and behavior of plasmoids. The electrodes of the button gun which was used in the present experiments were made of pure copper whereas the original button gun used by Bostick was titanium seeded with deuterium.<sup>1</sup> Because of the appearance of  $\rho$ , the mass per unit length in Eqs. (11) and (13), it would appear that the composition of the electrodes will affect the properties of a plasmoid although the measuring techniques which are presently available may not be sufficiently sensitive to enable a systematic investigation to be undertaken. Again, the higher ambient pressure at which Bostick<sup>1</sup> performed his pioneering experiments hardly seems likely to influence the velocity measurements significantly because of the very long mean-free paths in both cases.

Although the source capacitance in the present measurements is nearly twice that used by Bostick<sup>1</sup> and Harris *et al.*,<sup>2</sup> and the source voltage range correlates closely with that reported in this paper, it already has been shown both in this paper and elsewhere<sup>2,5</sup> that it is the surge current which is the significant parameter controlling the physics and, therefore, the differences in capacitors and source voltages are not really important.

### B. The Probe

We suggest that the probe which is described in this paper is more general in application than that used by Bostick<sup>1</sup> and Harris et al.,<sup>2</sup> which measured arrival times of plasmoids by recording the polarization field induced in the plasmoids by virtue of their motion across externally induced magnetic fields. However, although Harris et al.2 reported measurements of plasmoid velocity as a function of source voltage in magnetic-field-free space, it is not clear how these velocities were measured with a probe whose correct functioning seems to depend on the presence of externally induced magnetic fields. On the other hand, the probe which is used in the present investigation is well suited to measuring plasmoid velocities whether a magnetic field is present or not, since its functioning depends on a property of the plasmoid (i.e., its conductivity) rather than on its interaction with magnetic fields.

#### C. Subject of Measurement

The measurements which have been described in this paper are more general than those which have been described in the literature. We have attempted a description of the behavior of plasmoids in any direction in the plane of the current discharge, whereas measurements by Bostick<sup>1</sup> and by Harris *et al.*<sup>2</sup> were primarily oriented toward studying the speeds of plasmoids directly away from their button source. It should be borne in mind, though, that when the pioneering experiments were being performed it was still believed that the plasmoids moved through space as little isolated loops of plasma so that no necessity presented itself for the more general type of experiment which has been described in this paper.

#### D. The Creation Model

The toroidal model which has been described and developed in the present work is, of course, not original, but is merely a modification of the model first proposed by  $Bostick^1$  to explain the oscillographic signals which

were caused by the interaction of the plasmoids with probes in the presence of externally induced magnetic fields. However, the implications of a toroidal plasmoid in magnetic-field-free space has not previously been discussed in any detail, but rather was proposed as an intuitively obvious concept. In contrast, the approach which was used in the present work was to start with three *a priori* assumptions from which a self-consistent creation model of the plasmoid was deduced. This model showed itself capable not only of explaining experimental results similar to those recorded by Harris *et al.*<sup>2</sup> on the forward motion of plasmoids, but also those obtained by examining the plasmoid motion in the plane of the current discharge.

The main qualitative difference between the present physical picture and that used by Bostick<sup>1</sup> is that of the plasmoid shape after detachment from the buttongun electrodes. In this paper, no assumption has been made about the geometry assumed by the plasmoid after detachment from the button-gun electrodes. The plasmoid is regarded as a torus while the surge current is flowing and, when the latter has dropped to zero, each point on the plasma loop continues to move with a speed which is determined by its position on the plasma loop by virtue of the inertia of the plasma and the absence of other influences such as an externally induced magnetic field. On the other hand, Bostick had suggested that the plasma loop closes up on itself, and retains its toroidal shape as it moves away from the button gun. However, in view of the fact that the mechanism of projection is that of a uniform magnetic pressure which tends to increase the radius of the plasma loop, it is difficult to visualize a physical process which would cause the loop to close up on itself.

#### E. The Forward Motion of Plasmoids

The toroidal model of the plasmoid which has been presented in this paper has been successful in giving an interpretation of the forward motion. This model has revealed that the plasmoid speed directly away from the button gun is proportional to the average surge current. It is this fact that makes impossible any attempt at comparing the present results with those of Harris *et al.*<sup>2</sup> who reported only the source voltage without any mention of how the current in his experiments depended on the source voltage. All that can be said, therefore, is that an order of magnitude agreement exists within a similar range of source voltage.

#### F. Plasmoid Mass

One of the really new facts which this paper has brought to light is the constant line density of the plasmoid which is revealed in Table I to be  $10^{-9}$  kg/m to within a standard deviation of about 16%. From this, it was found possible to predict the dependence of the plasmoid mass on the average surge current and the predicted value has been verified experimentally to lie between 0.05 and 0.2  $\mu$ g for the source voltage range of interest. Bostick has stated that the mass of the plasmoids he studied was about 0.1  $\mu$ g,<sup>10</sup> but he has not explained how this figure was arrived at, and therefore, comparison between his method of estimation and technique reported in this paper is impossible.

## G. The Motion of Plasmoids in the Plane of the Discharge

The investigation of the polar profiles of plasmoids by probing techniques has not been reported in the literature and hence the present results cannot be compared with any other.

## VI. CONCLUSIONS

We have developed a creation model of a Bostick plasmoid and this model was developed on the basis of three a priori assumptions as to its properties in the hope that the behavior of our model would help us to understand the fundamental physics of plasmoids. It was hoped that at best the model would allow some crude order of magnitude estimates to be made of a few of the more interesting of the experimental results. In fact, without a single exception all three of the properties of the model which were guessed at have been fully substantiated by careful experimental readings, and the model itself has been successful not only in explaining and interpreting the experimental velocity measurements on the forward motion, but has also shown itself capable of predictions that have been verified by experimental measurement. The most surprising of the predictions of our theoretical analysis was the revelation that the mass of a plasmoid could be predicted in terms of experimental parameters by a method which hitherto is unrecorded in the literature and the prediction was verified by an experimental technique which also has not been recorded. We, therefore, conclude this paper by suggesting that the toroidal model which we have developed is probably a close approximation to the truth.

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