

Journal of Nuclear Science and Technology

ISSN: 0022-3131 (Print) 1881-1248 (Online) Journal homepage: www.tandfonline.com/journals/tnst20

Interference Phenomena of Neutron Waves in Heterogeneous Multiplying Systems

Takaaki MATSUMOTO

To cite this article: Takaaki MATSUMOTO (1973) Interference Phenomena of Neutron Waves in Heterogeneous Multiplying Systems, Journal of Nuclear Science and Technology, 10:3, 146-154, DOI: 10.1080/18811248.1973.9735396

To link to this article: https://doi.org/10.1080/18811248.1973.9735396

Published online: 15 Mar 2012.

Submit your article to this journal

Article views: 159

View related articles

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=tnst20

Journal of NUCLEAR SCIENCE and TECHNOLOGY, 10(3), p. 146~154 (March 1973).

Interference Phenomena of Neutron Waves in Heterogeneous Multiplying Systems

Takaaki MATSUMOTO

*Research Reactor Institute, Kyoto University**

Received July 26, 1972

An experimental and theoretical study has been conducted on two interference effects of neutron waves in heterogeneous multiplying systems—the interference between elementary waves having greatly different characteristics of propagation and the interference due to lattice effect, taking as example a plate-type system consisting of natural uranium and graphite. The experiments on neutron wave propagation were performed with external modulated sources. The effects examined on the neutron waves covered the multiplicity of system, the direction of propagation and the source spectrum. Analyses by fission expansion method have proved the existence of the first-mentioned form of interference in the source transient region. The second form of interference effect on the dispersion law with account taken of heterogeneity, is discussed numerically in comparison with the homogeneous multiplying system.

KEYWORDS: ***heterogeneous multiplying system, neutron waves, wave propagation, interference, reactor lattice, plate, natural uranium, graphite moderators, multiplicity, neutron spectra, fission expansion method, dispersion laws***

I. INTRODUCTION

Experiments on neutron wave propagation, which can be considered complementary to pulse die-away and exponential experiments, are effective in obtaining useful information on a system. A number of reports[1)~3)] have in recent years been published covering such experiments and dispersion laws derived in heterogeneous multiplying systems. In particular, the behavior of phase lag, which is peculiar to the neutron wave experiment and which is subject to various types of interference effect, are of interest from the standpoint of neutron wave optics as well as of the space dependent kinetics of reactors.

In heterogeneous multiplying systems, more particularly, in contrast to homogeneous non-multiplying systems, the following two interference effects must be expected to come into play: (1) that of different elementary waves having distinct characteristics of propagation and (2) that of lattice. The first cited effect may take place between the elementary wave excited by thermal neutrons having experienced slowing-down *via* fission processes (which we shall call slowing-down wave) and that excited by thermal neutrons having experienced only thermal scattering (thermal scattering wave). The thermal scattering wave is already familiar in non-multiplying systems, and the slowing-down wave which is dominant in the multiplying system has a more moderate lag of phase because of its large velocity in the slowing-down process. The lattice effect is due to the regular spatial arrangement of fuels. This interference is analogous to X-ray diffraction in a crystal. Quddus *et al.*[4)] have discussed the lattice effect on an axial wave in a rod system.

In the present paper, the interference effect between the slowing-down wave and thermal scattering wave, and the lattice effect are investigated in the source transient region and in the asymptotic region respectively. For reasons of simplicity, plate-type systems consisting of natural uranium and graphite shall be taken up. In the next chapter, propagation experiments using a thermal modulated source are described, and analyses are conducted by means of the fission expansion method based on the age-diffusion model, to prove the existence of the interference phenomena between the slowing-down wave and thermal scattering wave. In the third chapter,

* *Kumatori-cho, Osaka.*

dispersion laws for the perpendicular propagation are derived with consideration given to the heterogeneity, and the lattice effect on the wave in the asymptotic region is discussed in comparison with the homogeneous multiplying system.

II. Interference Phenomena in Source Transient Region

The interference phenomena between the slowing-down and the thermal scattering waves are examined in this chapter, both experimentally and theoretically. The experiments on interference have been performed in the plate-type lattice system of natural uranium and graphite with use made of a thermal modulated source. The experiment is compared with analysis by means of the fission expansion method based on the age-diffusion model. The effect of the external source spectrum is described in the last section.

1. Experiment

The interference phenomena of neutron waves were experimentally studied in two plate-type systems consisting of natural uranium and graphite. One system consists of 5 mm thick natural uranium and 200 mm thick graphite layers (referred to as the "C-U 5 system"), and the other system of 1 mm thick natural uranium and 100 mm thick graphite layers (the "C-U 1 system"). Both systems are prism-type piles, with cross sections of $100\times 100\,\mathrm{cm}^2$ and overall lengths of about 100 cm. A wave propagating in an arbitrary direction in a plate-type system is, in general, represented by the superposition of two fundamental waves *i.e.*, parallel and perpendicular. Hence, measurements have been made both in the parallel and perpendicular directions in the plate-type system. For the sake of comparison, experiments in a system consisting solely of graphite have also been made (the "C system": a 100 cm cube).

The arrangement of the present experiments is similar to those described in Ref. (5). A thermal neutron beam from a thermal column of the Kyoto University Reactor (KUR) is modulated sinusoidally by rotating slits of Cd (20 cycle/rotation). Before this modulated beam is applied to the system, an isotropic modulated source constituted of a graphite scatterer (a 10 cm cube) is installed at the center of the front surface of the system. A BF_3 counter (1″ diam.) is used to detect the thermal neutron wave propagating in the system. The temporal behavior of the waves along the axis of the prism system is recorded by a multi-channel pulse height analyzer. The behavior of the waves is fitted to a function $f(t)=A\cos(\omega t+\theta)+B$ by the method of least squares, whence the amplitude A and the phase lag θ are obtained as a function of the angular frequency ω and spatial position. Furthermore a modified source having a spectrum akin to the asymptotic one is used in the C-U 5 system, in order to observe the effect of the source spectrum on the transient region. Such a source is constituted by a spectrum shifter consisting of natural uranium and beryllium, positioned between the system and graphite scatterer.

2. Analysis by Fission Expansion Method

The time dependent thermal neutron flux $\phi(\boldsymbol{x}, t)$ in the moderator is expressed in the age-diffusion model by:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\boldsymbol{x}, t)=D\nabla^2\phi(\boldsymbol{x}, t)-\Sigma_a\phi(\boldsymbol{x}, t)+Q(\boldsymbol{x}, t)+\delta(\boldsymbol{x})S(t), \quad (1)$$

where

$$Q(\boldsymbol{x}, t)=\int_U d\boldsymbol{x}'\int_{-\infty}^{\infty} dt'\, p\cdot q(\boldsymbol{x}'\to\boldsymbol{x}:\ t-t')\cdot \nu\Sigma_f u(\boldsymbol{x}')\phi_f(\boldsymbol{x}', t'), \quad (2)$$

$\phi_f(\boldsymbol{x}', t')$: Thermal neutron flux in fuel
$u(\boldsymbol{x}')$: Distribution function of fuel
$q(\boldsymbol{x}'\to\boldsymbol{x}: t-t')$: Slowing-down kernel
$S(t)$: External source of thermal neutrons
p: Resonance escape probability
Σ_f: Macroscopic cross section for fission of fuel
ν: Average number of released neutrons per fission
D: Effective diffusion coefficient of moderator
Σ_a: Effective cross section for absorption of moderator
U: Total volume of fuel.

The effects of delayed neutrons are neglected because of the difference of the frequency range as well as because of their low intensity.

Now let us use the fission expansion method[6], in which the neutron flux is expanded by the number of fissions experienced:

$$\phi(\boldsymbol{x}, t)=\sum_{n=0}^{\infty}\phi^{(n)}(\boldsymbol{x}, t), \quad (3)$$

where a partial flux $\phi^{(n)}(\boldsymbol{x}, t)$ is contributed by neutrons having experienced n times fission in the system. The series (3) clearly converges for a subcritical system, and will converge rapidly in a source transient region where partial fluxes $\phi^{(n)}$ with small n are dominant. Inserting Eq. (3) into Eq. (1), we obtain, after Fourier transformation with respect to t,

$$D\nabla^2\phi^{(0)}(\boldsymbol{x}, \omega)-\left(\Sigma_a+\frac{j\omega}{v}\right)\phi^{(0)}(\boldsymbol{x}, \omega)+\delta(\boldsymbol{x})S(\omega)=0 \quad (4)$$

and

$$D\nabla^2\phi^{(n)}(\boldsymbol{x}, \omega)-\left(\Sigma_a+\frac{j\omega}{v}\right)\phi^{(n)}(\boldsymbol{x}, \omega)+\lambda_0\int_U d\boldsymbol{x}' q(\boldsymbol{x}'\to\boldsymbol{x}:\omega)\cdot u(\boldsymbol{x}')\phi_f^{(n-1)}(\boldsymbol{x}', \omega)=0 \quad (5)$$

for $n=1, 2, \ldots$, where $\lambda_0=p\nu\Sigma_f$.

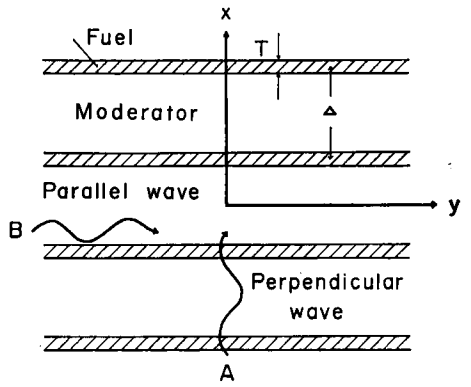


Fig. 1 Plate-type system

In the case of perpendicular propagation, indicated by A in **Fig. 1**, we should put

$$u(\boldsymbol{x})=u(x)=\begin{cases}1, & \text{for } x_i-T/2\leqslant x\leqslant x_i+T/2\\ 0, & \text{otherwise,}\end{cases} \quad (6)$$

where T: Thickness of a fuel plate
x_i: Central position of the i-th fuel plate.

While, strictly speaking, the waves have transverse propagating components because of the finite cross section of the prism system, they can be expected to be negligibly small. The neutron wave in the perpendicular propagation $\phi_\perp$ is expressed with use made of the modal representation of the transverse distribution:

$$\phi_\perp^{(n)}(\boldsymbol{x}, \omega)=\sum_{lm}\psi_{lm}(y, z)\phi_{\perp lm}^{(n)}(x, \omega) \quad (7)$$

for $n=0, 1, \ldots$, where

$$\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi_{lm}(y, z)=-B_{\perp lm}^2\psi_{lm}(y, z)$$

$$B_{\perp lm}^2=\left(\frac{2l+1}{a}\pi\right)^2+\left(\frac{2m+1}{b}\pi\right)^2,$$

a, b: Dimensions of cross section of prism including extrapolated distance.

Integration of Eqs. (4) and (5) multiplied by a complex conjugate of $\psi_{lm}(y, z)$ is carried out with Eqs. (6) and (7), and we obtain

$$D\frac{d^2}{dx^2}\phi_{\perp lm}^{(0)}(x, \omega)-\left(\Sigma_a+DB_{\perp lm}^2+\frac{j\omega}{v}\right)\phi_{\perp lm}^{(0)}(x, \omega)+\delta(x)S_{lm}(\omega)=0 \quad (8)$$

and

$$D\frac{d^2}{dx^2}\phi_{\perp lm}^{(n)}(x, \omega)-\left(\Sigma_a+DB_{\perp lm}^2+\frac{j\omega}{v}\right)\phi_{\perp lm}^{(n)}(x, \omega)+\lambda\sum_i q_{lm}(x_i\to x:\omega)\phi_{\perp lm}^{(n-1)}(x_i, \omega)=0, \quad (9)$$

where

$$\lambda=\lambda_0 T/\zeta=p\nu\Sigma_f T/\zeta$$

$$q_{lm}(x_i\to x:\omega)=e^{-B^2{}_{\perp lm}\tau-j\omega l_s}\frac{e^{-(x-x_i)^2/4\tau}}{\sqrt{4\pi\tau}}$$

ζ: Disadvantage factor
τ: Fermi age of thermal neutron from fission energy
l_s: Slowing-down time from fission to thermal energy.

Here the flux ϕ_f in the thin fuel plate has been replaced by ϕ/ζ with the flux ϕ in the moderator and the disadvantage factor ζ[7]. We obtain solutions for the partial waves from Eqs. (8) and (9):

$$\phi_{\perp lm}^{(0)}(x, \omega)=c_{lm}e^{-\rho_{lm}x} \quad (10)$$

and

$$\phi_{\perp lm}^{(n)}(x, \omega)=\lambda\sum_i G_{lm}(x_i\to x:\omega)\cdot\phi_{\perp lm}^{(n-1)}(x_i, \omega), \quad (11)$$

for $n=1, 2, \ldots$, where

$$\rho_{lm}=\left[\frac{\Sigma_a}{D}+B_{\perp lm}^2+\frac{j\omega}{vD}\right]^{1/2} \quad (12)$$

$$G_{lm}(x_i\to x:\omega)=\int_{-\infty}^{\infty}dx'\frac{e^{-\rho_{lm}|x-x'|}}{2\rho_{lm}D}q_{lm}(x_i\to x':\omega), \quad (13)$$

c_{lm}: Coefficient determined by source condition.

Here Eq. (12) corresponds to a dispersion law of

the thermal scattering wave, where the effects of the absorption of the fuel plates are alone considered (in Σ_a and D).

In the case of parallel propagation, indicated by B in Fig. 1, we also obtain solutions for the partial waves by the modal representation of the transverse distribution similarly to the case of perpendicular propagation: A solution for $n=0$ is identical to Eq. (10), and for $n=1, 2, \ldots$, the solutions for the l and m mode are given by

$$\phi^{(n)}_{/\!/lm}(y, \omega)=\lambda \sum_{il'} \psi_{l'}(x_i)\psi_l^*(x_i) \cdot \int_{-\infty}^{\infty} dy' G_{lm}(y' \to y : \omega)\phi^{(n-1)}_{/\!/l'm}(y', \omega), \quad (14)$$

where $\psi_l^*(x)$: Complex conjugate of the spatial mode function $\psi_l(x)$.

3. Comparison of Experiment and Theory

The amplitude A and phase lag θ are shown respectively in **Figs. 2** and **3** for the perpendicular propagation experiment in the C-U 5 system using the thermal modulated source, which are derived by least squares fitting to $f(t)=A\cos(\omega t+\theta)+B$. While the relative amplitude decreases straightforwardly with angular frequency at each measuring position, the phase lag behaves in a fairly complex pattern at positions removed from the source. **Figures 4** and **5** show the dependence of the amplitude and phase lag, respectively, on the distance from source. The relative amplitude would appear to decay

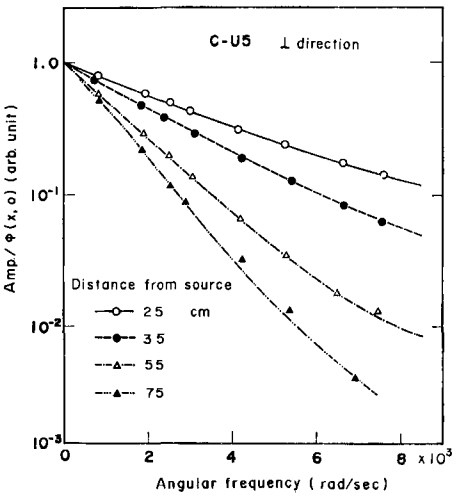


Fig. 2 Relative amplitude of perpendicular wave in C-U 5 system

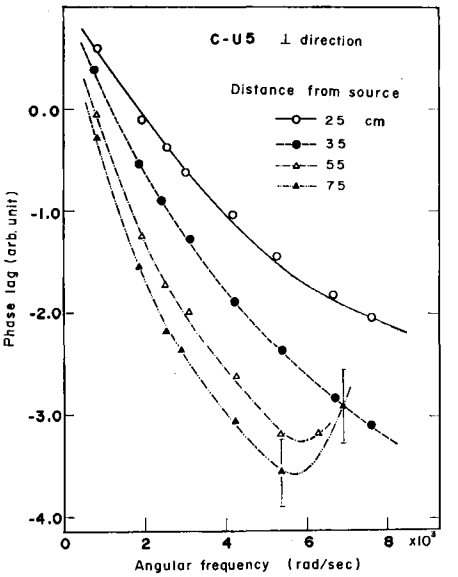


Fig. 3 Phase lag of perpendicular wave in C-U 5 system

exponentially, but the phase lag shows a nonlinear behavior. The decay of the steady flux ($\omega=0$) is also non-exponential, as shown in Fig. 1 in Ref. (6).

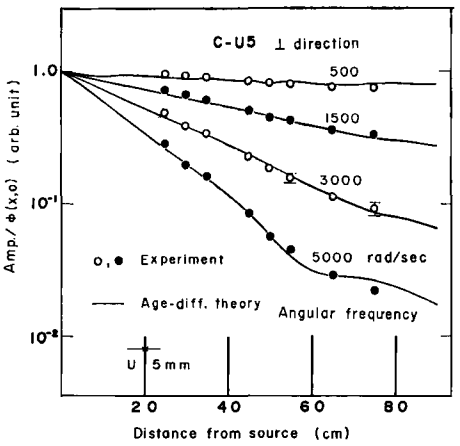


Fig. 4 Relative amplitude of perpendicular wave in C-U 5 system

The perpendicular propagation observed in the C-U 1 system is qualitatively similar to the case of the C-U 5 system. Comparison between the two systems for the amplitude and phase lag are shown in **Figs. 6** and **7** respectively, in the typical case of $\omega=3{,}000$ rad/sec. The parallel wave is also presented in the same figures. The

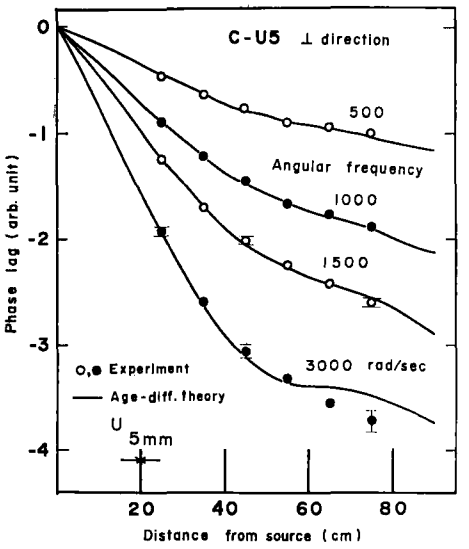


Fig. 5 Phase lag of perpendicular wave in C-U 5 system

rate of decay of the relative amplitude is smaller and the divergence from linearity of the phase lag is greater in the C-U 5 system than in the C-U 1, due to the larger multiplicity of the former system. Furthermore the phase lag depends on the direction of propagation more sensitively than the amplitude. The non-linearity of the phase lag is larger in perpendicular than in parallel propagation. Contrary to the multiplying system, the neutron wave measured in the C system is precisely represented by a plane wave.

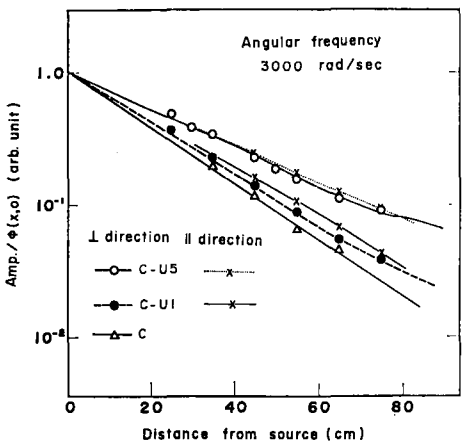


Fig. 6 Comparison of relative amplitude

Before we analyze these phenomena observed in the heterogeneous multiplying system, let us

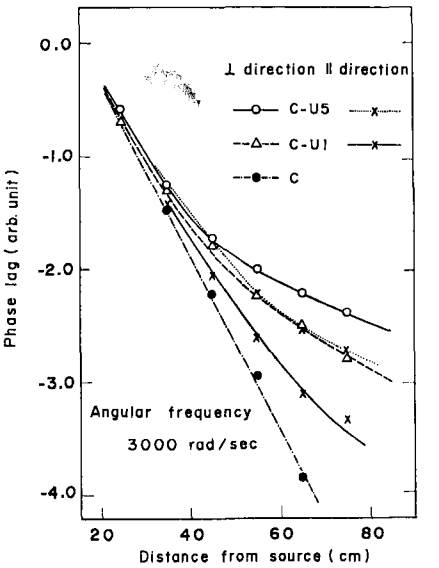


Fig. 7 Comparison of phase lag

first consider how the slowing-down wave propagates in the moderator. The propagating characteristics of the slowing-down wave are given by the function G in Eq. (13) on the age-diffusion model. **Figures** 8 and **9** show the propagating characteristics of the slowing-down wave in the graphite moderator. Also shown in the same figures are the characteristics of the thermal scattering wave, calculated from exp $(-\rho_{00}x)$ by using Eq. (12). The decay rate of

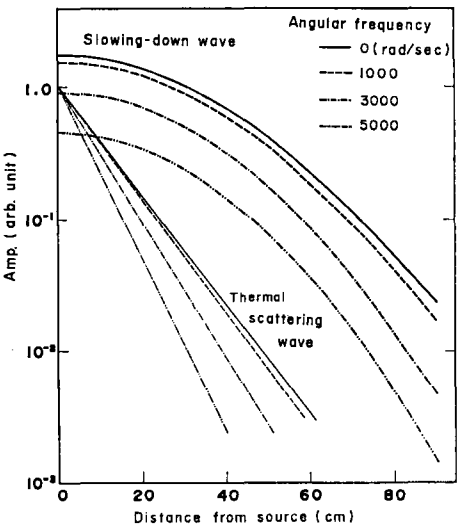


Fig. 8 Comparison of amplitude between slowing-down wave and thermal scattering wave

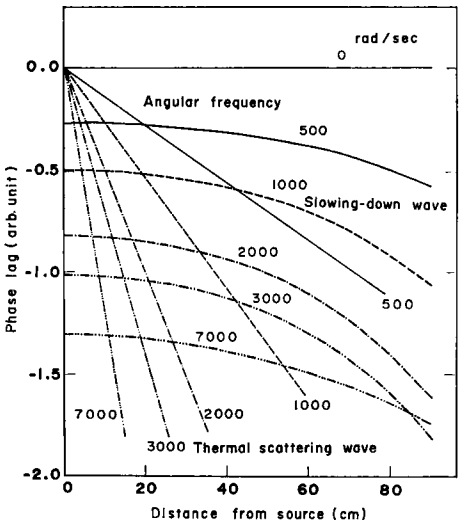


Fig. 9 Comparison of phase lag between slowing-down wave and thermal scattering wave

the slowing-down wave is small in the neighborhood of source because of the large age of graphite, and becomes comparable to that of the thermal scattering wave in locations removed from source. On the other hand, on the phase lag, the effect of fission is quite marked: the large velocity of the slowing-down neutrons makes the phase lag of the elementary waves fairly small. Thus the neutron waves in a multiplying system will behave in a complex manner due to interference between the elementary waves.

The fission expansion method mentioned in the preceding chapter, which deals with each slowing-down wave separately, should be effective in studying these complex phenomena of interference. Numerical calculations of the partial waves propagating perpendicularly have been performed by using Eqs. (10) to (13). Then the total flux was calculated with the aid of Eq. (3) with error smaller than 1%. This condition was satisfied by retaining only the first several terms of the partial flux. A spatial fundamental mode ($l=m=0$) was alone considered in the calculations because of the large decay rate of the higher modes. The calculated results for the amplitude and phase lag are shown in Figs. 4 and 5 respectively for perpendicular propagation in the C-U 5 system, in obtaining which curves, the constants given in **Table 1** were used. The values of experiment

Table 1 Constants used in calculations

$\tau=350\ \mathrm{cm}^2$
$B_1^2=0.00184\ \mathrm{cm}^{-2}$
$D=0.844\ \mathrm{cm}$
$\sqrt{\Sigma_a/D}=0.095\ \mathrm{cm}^{-1}$
$\lambda=0.111$

and calculation are in good agreement for the total wave. The amplitude of the partial waves is shown in **Fig. 10** for the case of $\omega=3{,}000$ rad/sec. The phase lag of the total wave changes linearly with distance in the neighborhood of the source, where the thermal scattering wave or the incident wave $\phi^{(0)}$ is predominant. The phase lag then becomes non-linear as a result of the interference between $\phi^{(0)}$, $\phi^{(1)}$ and the higher order waves. The parallel propagating waves can be described in a similar manner, but the interference effect is smaller in this case because of the continuous distribution of fission sources (as indicated by Eq. (14)).

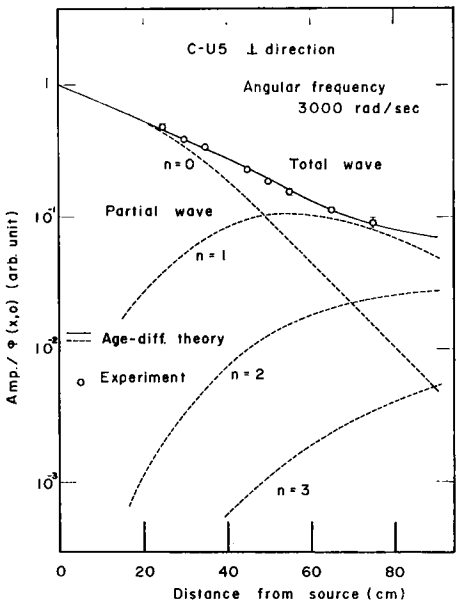


Fig. 10 Contribution of partial waves

4. Effect of Source Spectrum

Analysis by the fission expansion method has proved that in the experiment using a thermal source there exists a wide source transient region, predominated by neutrons having experienced

fission few times. The spread of the transient region depends on the multiplicity of the system, the Fermi age of the moderator and the spectrum of the external source. An experiment with a modified source possessing a spectrum akin to the asymptotic was performed in order to study the effects of differences in the source spectrum on the transient region. **Figure 11** shows the behavior of the perpendicular wave in the C-U 5 system at $\omega=3{,}000$ rad/sec using this modified source. Compared with the case of thermal source, the relative amplitude and steady flux appear to decay more closely to exponential in the smaller distances. The phase lag, on the other hand, shows a small oscillation in a wide range of distances. This fact suggests that a completely asymptotic state has not been attained under the present measuring conditions. Another conclusion that can be drawn from the foregoing observations is that the phase lag is an integral quantity sensitive to the spectrum in the system. Thus an asymptotic region will be made by better adjustment of the source spectrum with attention given to the phase lag.

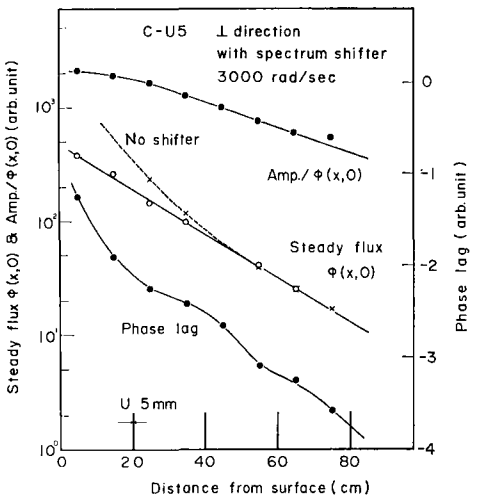


Fig. 11 Effects of source spectrum

III. Interference by Lattice in Dispersion Law

In the foregoing chapter, we have examined the interference effects between the thermal scattering wave and the slowing-down wave, and have seen that the effets of the lattice were reduced by the strong intensity of the thermal source used. In the asymptotic region, however, where the thermal scattering wave has completely decayed, the effects of the lattice may be enhanced at frequencies high enough to approach the neutron wave length to the fuel spacing.

In this chapter, we shall consider the interference effect of the lattice on the asymptotic behavior of the wave *i.e.*, on the dispersion law. Perpendicular propagation in the plate-type system is considered because this is the case where the interference effect by lattice may be the most pronounced. We use the age-diffusion model whose validity has been proved in the foregoing chapter by the good agreement obtained between experiment and calculation. First, we derive dispersion laws for the perpendicular propagation in a plate-type system. The lattice interference effect on the dispersion law is discussed in comparison with the homogeneous multiplying system. Finally the relation between the fission expansion method and the dispersion law is considered.

1. Dispersion Law

The neutron wave propagating perpendicularly in the plate-type system is expressed in the asymptotic region by letting $S(t)\to 0$ in Eq. (1):

$$D\frac{d^2}{dx^2}\phi_{\perp lm}(x,\omega)-\left(\Sigma_a+DB^2_{\perp lm}+\frac{j\omega}{v}\right)\phi_{\perp lm}(x,\omega)+\lambda\sum_i q_{lm}(x_i\to x:\omega)\phi_{\perp lm}(x_i,\omega)=0. \tag{15}$$

We now assume the separability of a macroscopic propagating wave and the local neutron flux distribution, *i.e.*,

$$\phi_\perp(x,\omega)\sim f(x)e^{-\rho_{as}x}, \tag{16}$$

where $\rho_{as}=\alpha+j\beta$, and $f(x)$ is the distribution of the neutron flux in a unit lattice. Inserting Eq. (16) into Eq. (15), we operate $\int_{-\infty}^{\infty}dx e^{-j\frac{2k\pi}{\Delta}x}\cdot$, to obtain

$$\left[D\{(\rho_{as}-jB_k)^2-B^2_{\perp lm}\}-\left(\Sigma_a+\frac{j\omega}{v}\right)\right]\tilde{f}_k+\int_{-\infty}^{\infty}dx e^{-jB_kx}\cdot\lambda\sum_i q_{lm}(x_i\to x:\omega)\cdot f(x_i)e^{-\rho_{as}(x_i-x)}=0, \tag{17}$$

where $B_k=2k\pi/\Delta$ for $k=0, 1, \ldots$

$$\tilde{f}_k=\int_{-\infty}^{\infty}dx e^{-jB_kx}f(x)$$

Δ: Length of unit lattice.

Application of Poisson's sum formula[8] to the

third term in Eq. (17) gives

$$\frac{\lambda}{\Delta}\sum_{k'=-\infty}^{\infty}\int_{-\infty}^{\infty}dtf(t)e^{-j\frac{2\pi}{\Delta}(k+k')t}\cdot\int_{-\infty}^{\infty}dxe^{-jB_kx}q_{lm}(x:\omega)e^{\rho_{as}x}$$
$$=\frac{\lambda}{\Delta}\sum_{k'}\tilde{f}_{k+k'}\cdot\exp[\tau\{(\rho_{as}-jB_k)^2-B_{\perp lm}^2\}-j\omega l_s]. \quad (18)$$

From Eqs. (17) and (18) we obtain

$$a_k\tilde{f}_k+\sum_{k'=-\infty}^{\infty}\tilde{f}_{k+k'}=0, \quad (19)$$

where

$$a_k=\frac{1}{k_\infty}e^{-\tau\{(\rho_{as}-jB_k)^2-B^2_{\perp lm}\}+j\omega l_s}\cdot[L^2\{(\rho_{as}-jB_k)^2-B_{\perp lm}^2\}-1-j\omega t_s] \quad (20)$$
$$t_s=1/v\Sigma_a.$$

The condition that $\tilde{f}_k$ in Eq. (19) has non-trivial solutions is

$$|1+a_k\delta_{kk'}|=0 \quad \text{for } k, k'=0, 1, \ldots \quad (21)$$

Equation (21) is the dispersion law for the perpendicular wave in the plate-type system with the heterogeneity of the lattices taken into account.

2. Numerical Results

An approximation including only the $k=0$ term in Eq. (21) gives the dispersion law in the homogenized system:

$$1+a_0=0, \quad (22)$$

i.e.,
$$L^2(\rho_{as}{}^2-B_{\perp lm}^2)-1-j\omega t_s+k_\infty e^{\tau(\rho_{as}{}^2-B^2_{\perp lm})-j\omega l_s}=0. \quad (23)$$

Equation (23) is identical to the dispersion law derived by Kylstra *et al.*[9] Solutions (α, β) for the C-U5 system, which satisfy the real and imaginary parts of Eq. (23) at the same time, were obtained numerically at each frequency, as shown by the solid curve in **Fig. 12**. There is observed a maximum value of β, which owes its existence to the slowing-down wave, in marked contrast with the case of a non-multiplying system.

When the heterogeneity effect is approximated with terms up to $k=\pm1$ taken up, we obtain a modified dispersion law:

$$1+a_0+\frac{2a_0}{a_1}=0 \quad (24)$$

In this equation, the heterogeneity effect is presented explicitly by the third term. In the case of the C-U5 system, the contribution of the third term is negligibly small. In a system with $\Delta=100$ cm but with the other constants held the same as in the C-U5 system, the numerical results of the dispersion law from Eq. (24) are shown by the broken curve in Fig. 12. The homogenized dispersion law for the present system is the same as the solid curve in Fig. 12, since the term $1+a_0$ does not include the parameter Δ. Therefore the difference between the solid and broken curves in Fig. 12 corresponds to the interference effect of the lattice in the system.

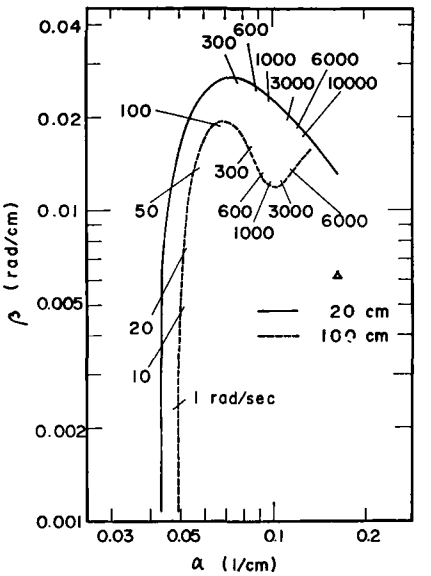


Fig. 12 Effects of lattice on dispersion law

3. Fission Expansion Method in Dispersion Law

In Chap. II, we have proved that the fission expansion method is useful for analyzing the transient region. We shall now consider an application of the fission expansion method to the dispersion law. A homogeneous system will be taken up on account of its simplicity, in which the summation in Eq. (9) is replaced by integration. Fourier transformation of the equation for x gives

$$\tilde{\phi}^{(n)}(k,\omega)=\int_{-\infty}^{\infty}dxe^{-jkx}\phi^{(n)}(x,\omega)=\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\tilde{\phi}^{(n-1)}(k,\omega)=\cdots=\left[\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right]^n\tilde{\phi}^{(0)}(k,\omega), \quad (25)$$

where $\tilde{q}(k:\omega)$: Fourier transformation of $q(x:\omega)$.

The total wave is given from Eq. (3):

$$\tilde{\phi}(k,\omega)=\int_{-\infty}^{\infty}dx e^{-jkx}\phi(x,\omega)=\sum_{n=0}^{\infty}\tilde{\phi}^{(n)}(k,\omega)$$
$$=\left[1+\left\{\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right\}+\left\{\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right\}^2+\cdots\right]\tilde{\phi}^{(0)}(k,\omega)$$
$$=\left[1-\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right]^{-1}\tilde{\phi}^{(0)}(k,\omega). \quad (26)$$

The equation $1-\lambda\tilde{q}(k:\omega)/D(k^2+\rho^2)=0$, which gives the dispersion law in the homogeneous multiplying system, is the same as Eq. (23). Therefore, application of the fission expansion method to the dispersion law means the following expansion:

$$\left[1-\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right]^{-1}$$
$$=1+\left\{\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right\}+\left\{\frac{\lambda\tilde{q}(k:\omega)}{D(k^2+\rho^2)}\right\}^2+\cdots. \quad (27)$$

Equation (27) may be a useful approximate expression for the disparsion law in a system with small multiplicity, where the expansion converges repidly.

IV. Conclusion

Two interference effects of the heterogeneous multiplying system on the neutron wave have been examined for the case of a plate-type system, chosen as example for its simplicity, and consisting of natural uranium and graphite. The existence of interference phenomena between elementary waves differing greatly in their characteristics of propagation has been proved both by experiment using a thermal modulated source and by analysis based on the fission expansion method. The results show that the dependence of propagating characteristics for the elementary wave on the experienced process, especially of the phase lag, is strong in a multiplying system, and that there exists a wide source transient region in the experiment with the thermal source. Furthermore the experiment with modified source suggests that the phase lag is an integral quantity sensitive to the neutron spectrum in the system.

The other interference effect, *i.e.*, the lattice effect, was studied theoretically, based on the age-diffusion model, those validity was proved by experiments in the transient region. The dispersion law has been derived for the perpendicular wave in the plate-type system, and the lattice effect on the dispersion law studied numerically. It has been shown that marked interference effects exist in some systems.

Acknowledgment

The author is grateful to Prof. T. Shibata for his encouragement. He also thanks Dr. M. Utsuro for useful discussions, and Messrs. Y. Ishikawa and K. Okumura for their assistance with the instrumentation.

References

(1) Dunlop, J. H., Uhrig, R. E.: *Symp. Neutron Noise, Waves & Pulse Propagation*, Florida, (1966).
(2) Sumita, K., Takahashi, A.: *Japan-U. S. Seminar Nucl. Reactor Noise Analysis*, Tokyo & Kyoto, (1968).
(3) Wada, M., Utsuro, M.: *Annu. Rep. Res. Reactor Inst., Kyoto Univ.*, **3**, 107 (1970).
(4) Quddus, M. A., Cochran, R. G., Emon, D. E.: *Nucl. Sci. Eng.*, **35**, 342 (1969).
(5) Utsuro, M., Inoue, K., Shibata, T.: *J. Nucl. Sci. Technol.*, **5**(6), 298 (1968).; Utsuro, M., Matsumoto, T., Ishikawa, Y., *et al.*: *Tech. Rep. Res. Reactor Inst., Kyoto Univ.*, **26**, (1966).
(6) Matsumoto, T.: *J. Nucl. Sci. Technol.*, **7**(9), 42 (1970).
(7) Galanin, A. D.: "*Thermal Reactor Theory*", (1960), Pergamon Press, New York, London.
(8) Courant, R., Hilbert, D.: "*Method of Physical Mathematics*", (Translated into Japanese), (1968), Tokyo Tosho.
(9) Kylstra, C. D., Uhrig, R. E.: *Nucl. Sci. Eng.*, **22**, 191 (1965).